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UNSOLVED PROBLEMS OF COSMICAL PHYSICS.*

BY

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I HAVE heard it said that Rowland, when asked for advice by a student whose apparatus had failed to give the expected result would reply "Do something to it until something happens." Every experimentalist will recognize the wisdom of the counsel; but, alas, the student of cosmical physics is unable to avail himself of it to any great extent. There is little that he can do to much of the apparatus with which he deals, to the sun, the stars, and the planets: He can only wait and see what happens. How he would like to inclose our globe and its atmosphere in a lead casing several metres thick, and see what change is thereby produced in atmospheric electric phenomena, or reverse the direction of rotation of the earth, and study the changes in the earth's magnetic field for a few million years. As he would find it difficult to discover anyone willing to finance either of these projects, however, there is nothing for him to do but wait, observe what he can, duplicate nature's processes, as best he may, on a small scale in his laboratory and above all things, think hard.

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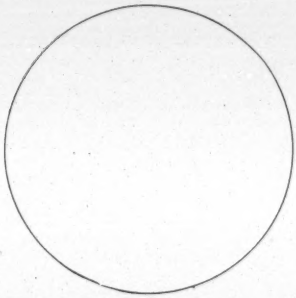
ATMOSPHERIC ELECTRICITY.

Fig. 1 represents a circle. The thickness of the line is not great, yet that line represents in relation to the size of the earth as represented by the circle, a shell seven times as thick as that to which practically all of our atmospheric electric observations are confined. Within this region we know that there is an electric field which is perpendicular to the surface of the earth, and that it is brought about by a negative charge on the surface and a positive charge in the atmosphere. The field amounts to about 150 volts per metre at the surface, and goes through fairly regular vari-

FIG. 1.



ations throughout the day, and throughout the year, changes of the order of 50 per cent. or more of its value. It diminishes with altitude until at an altitude of about 10 kilometres it attains a value insignificant compared with that at the surface.

ATMOSPHERIC CONDUCTIVITY.

Now the atmosphere is a very feeble conductor of electricity, so that the positive charge in the atmosphere tends to become neutralized by the negative charge on the earth. The conductivity of the atmosphere is produced by the presence in it of charged molecules of nitrogen and oxygen, called ions. These, in virtue of their charge, move in an electric field and give up their charges to bodies which they encounter, thereby discharging them. Under ordinary conditions there are about 1000 of each sign of ions in

each cubic centimetre, a very small number compared with the 30 million million million molecules of air which that cubic centimetre contains. The positive ion is produced by detaching from the neutral molecule one of those electrons which modern physics has taught us to regard as one of the fundamental constituents of all matter. The negative electron attaches itself sooner or later to some neutral molecule and, thus loaded, constitutes the negative ion. The ejection of the electron from the atom is brought about by some external agency. Of these there are principally three kinds, all of which are represented in the radiations shot out from a substance like radium, which is in a state of continual spontaneous disintegration.

First there is the alpha particle, a positively charged atom of helium, moving with a speed of 18,000 miles per second. Then there is the beta ray, which is simply a negative electron moving with a speed approximating that of light. Finally we have the gamma ray, which is a kind of wave motion in the æther. When an alpha ray approaches an atom, it tends to snatch away an electron, just as a comet coming into our solar system might tend to snatch away the moon. The beta ray tends to hurl the electron out by repulsion, just as a comet fitted with a huge air blast might tend to blow the moon away. Finally the gamma ray acts by setting up a sort of agitation in the æther, in virtue of which the electron acquires a velocity great enough to enable it to leave the atom.

A large portion of the ionization, and consequently of the conductivity of the atmosphere is produced by the presence in it of radium emanation. A cubic centimetre of the atmosphere contains only about one and a half molecules of emanation while it contains about 30 million million million molecules of air; but, this emanation is nevertheless sufficient to contribute quite appreciably to the ionization. The alpha, beta and gamma rays from the radioactive emanations in the air, the gamma rays from the radium in the soil, and a possible penetrating radiation of cosmical origin constitute the main sources of ionization in the atmosphere. The fact that the ionization over the great oceans, where there is no radioactive material, is comparable with that over land, suggests that the cosmical radiation may play an important rôle, although recent experiments have thrown some doubt upon the reality of

this radiation, at least as regards its presence at low altitudes. Thus to sum up the situation, certain rays, alpha, beta, and gamma rays, and possibly certain cosmical rays, are responsible for the ejection of electrons from a few molecules of the air, thus resulting in the production of positive and negative ions, which render the atmosphere conducting.

The conductivity of the atmosphere at the earth's surface is extremely small compared with that of such a substance as copper. Thus a cylinder of air one inch long offers as much resistance to the flow of the electric current as would a copper cable, of equal cross-section, reaching from here to the star Arcturus and back twenty times over. Nevertheless, this conductivity is sufficient to insure that 90 per cent. of the earth's charge would disappear in ten minutes were there no means of replenishing the loss. The explanation of the maintenance of the earth's charge is the great problem of atmospheric electricity. The replenishment to be accounted for is not large. The total current from the whole of the earth's surface is only about 1000 amperes, or as much as is taken by 3000 incandescent lamps; but, our knowledge of the nature of electrical phenomena is sufficiently definite to place very critical restrictions upon any process which we may imagine for the replenishment.

THE MAINTENANCE OF THE EARTH'S CHARGE.

It is obvious that if negative electricity is leaving the earth, or positive electricity entering it in virtue of the potential gradient operating in a conducting atmosphere, any compensating stream of electricity must take place by processes which operate in opposition to the electrical forces of the field. The theories which have been proposed divide themselves for the most part into two classes, those in which gravity is the primary agent instrumental in moving the charges against the field, and those in which the return of negative corpuscles to the earth is brought about in opposition to the field through the agency of a very high velocity produced in them by some means or other.

Theories Invoking Gravitation.—As an example of the former class we have the theory of Mr. C. T. R. Wilson, to the effect that the replenishment takes place through the agency of rain. Theoreti-

cal considerations have been thought to suggest that the rain-drops should form on the atmospheric ions, and more copiously on the negative than on the positive ions, so that rain might be expected to be, on the whole, negatively charged. The charged drops, falling to earth under gravity, would do so in opposition to the electric field, and would constitute the replenishment. This theory is open to two primary objections. In the first place, while rain is charged, and to a degree probably sufficient to account for the necessary replenishment, it is found that 90 per cent. of the rain which falls is *positively* charged. Again, it appears that there are grave theoretical difficulties concerned with the possibility of condensation of water upon atmospheric ions, in the form of drops of appreciable size, so that this theory is now generally discarded.

A theory which came near to being successful, and which undoubtedly plays a part in the origin of the earth's charge is one due to Ebert. Ebert's theory, which constitutes a modification of an earlier theory due to Elster and Geitel, invokes the fact that if ionized air be passed through a fine tube, the negative ions diffuse to the walls of the tube more rapidly than do the positive ions, so that the air which emerges from the tube is positively charged. Ebert applies this to the atmospheric electric problem, by supposing that the air which is to be found in the interstices of the soil, and which is ionized by the radioactive material therein, is drawn out during the periods of falling barometric pressure, leaving an excess of negative charge on the walls of the interstices. The positive charge which emerges would be held in the immediate vicinity of the ground by the negative charge, but here Ebert invokes the aid of upward air currents, which carry it, against the field, into the higher regions of the atmosphere. The theory has been criticised on the basis that the emission of ions from the ground would be insufficient, and the upward currents too feeble. I think, however, that one of the most serious objections is to be found in a conclusion, which follows comparatively simply from theoretical considerations, to the effect that, on such a theory, it would result that, before the ascending positive charge had risen to an altitude of a kilometre or so, it would have disappeared almost completely, devoured as it were by the negative charge continually fed into it from the earth below, through the medium of the con-

ducting atmosphere.[1] We should obtain a positive charge in the atmosphere, a negative charge on the earth's surface, a conduction current and a potential gradient; but, all of these phenomena would be confined to a layer of the atmosphere about a kilometre or so in thickness. The whole of the positive charge in the atmosphere would be found in this layer, and, being equal to the negative charge on the earth's surface, since that was formerly its partner in neutrality, it would annul the field at all greater altitudes.

Certain other difficulties present themselves in the theory of Ebert, and in the precipitation theory of C. T. R. Wilson. Thus, in the case of both theories, the corresponding positive and negative charge densities would remain sensibly in the regions in which they originated, with the result that there would be no electric field at other parts of the earth. It is useless to invoke the winds for the transportation of the atmospheric positive charge to remote regions, for 90 per cent. of it would disappear in ten minutes in the course of its journey on account of the conductivity of the

[1] The reason is as follows: Let ρ be the density and v the velocity of the ascending positive electricity. Let X be the potential gradient, and λ the conductivity at a point in the atmosphere; and let the axis of x be taken vertically upwards from the earth's surface.

The equilibrium between the conduction and convection current at all altitudes requires that

$$\rho v + X\lambda = 0 \qquad (1)$$

Poisson's equation gives

$$\frac{\partial X}{\partial x} = 4\pi\rho \qquad (2)$$

so that, from (1) and (2),

$$\frac{v}{4\pi}\frac{\partial X}{\partial x} + X\lambda = 0$$

which gives

$$X = X_0 e^{-4\pi\int_0^h \frac{\lambda}{v}\,dx} \qquad (3)$$

where X is the potential gradient at the altitude h.

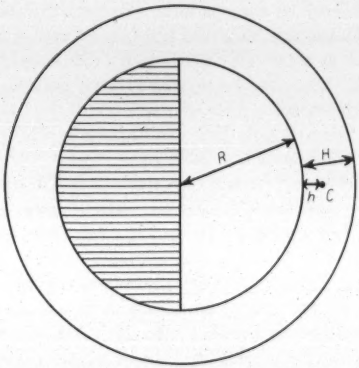
Now, since λ increases with altitude and v decreases, the value of X as given by (3) is certainly less than the value obtained by substituting in it the surface value of λ/v, *viz.*, λ_0/v_0.
Thus,

$$\frac{X}{X_0} < e^{-4\pi\frac{\lambda_0}{v_0}h}$$

If $\lambda_0 = 3 \times 10^{-4}$ e.s.u., and $v_0 = 100$ cm./sec., and if we put $h = 1000$ metres, we find that the value of X/X_0, at this altitude is less than 0.02.

atmosphere. Difficulties of this kind confront any theory in which the replenishment of the charge takes place by the separation of charges at isolated places in such a manner that the complementary positive and negative parts are to be found comparatively near together. They become much less serious if we adopt the likely hypothesis that the lower atmosphere is surrounded by the equivalent of a conducting shell. Such a shell seems necessary to account for the reflection of wireless waves around the earth, and there is strong reason to suppose its existence on the basis of our knowl-

FIG. 2.



edge of the mechanism of the variations in terrestrial magnetism, to which I shall presently refer.

In the presence of such a lawyer, the action of a charged cloud is to set up a difference of potential between the layer and the earth, which difference of potential is handed round, as it were, to all parts of the earth, in view of the fact that the difference in potential between the earth and the layer must be the same at all places. A detailed consideration of the action of such a layer shows[2] that, if a charged cloud C (Fig. 2) is to be found at an altitude h above the earth's surface, and if H is the altitude of the conducting layer, and R the radius of the earth, the hemisphere of the earth, which is symmetrically remote from the charged cloud (the shaded hemisphere in Fig. 2), would receive R/H

[2] The full theory of this matter will be given in a subsequent publication.

times the number of tubes of force which it would receive in the absence of the layer, and h/H times the number which it would receive if the charge on the earth and the charge on the cloud were pasted uniformly over the earth and atmosphere respectively. Thus, if rain should fall from a cloud at an altitude of 5 kilometres, and if the effective altitude of the conducting layer were 50 kilometres, the average contribution to the potential gradient on the shaded hemisphere would be one-tenth of what it would have been if the charges had been uniformly distributed over the earth and atmosphere, and over a hundred times as great as it would have been in the absence of the conducting layer. By the aid of such a conducting layer, we thus see how every centre of separation of charge, such as a rain storm, can pay a contribution, positive or negative as the case may be, to the general potential gradient at every part of the earth.[3]

However, even though difficulties attending the localization of the potential gradient be removed by the postulation of the conducting layer, there yet remain the others of which we have spoken, and which are sufficiently severe to make it improbable that the Ebert theory can figure as the main cause responsible for the earth's charge.

Corpuscular Theories.—Turning now to theories in which the replenishment of the charge comes about by the agency of high-speed electrified corpuscles shot into the earth, the first of these was proposed by G. C. Simpson. In this theory it was supposed that the sun emitted negative and positive corpuscles of high penetrating power. The former were supposed to pass right through our atmosphere and charge the earth, while the latter were of less penetrating power, and were caught in the atmosphere. In this way, the earth would continually receive negative and the atmosphere positive charge. The ordinary processes of atmospheric con-

[3] It may be remarked, moreover, that this action of the conducting layer minimizes slightly that objection to the Ebert theory which is founded upon the impossibility of the positive charge reaching altitudes greater than a kilometre or so. For, even such a small separation in the region of replenishment makes its own small contribution to the potential gradient at other places, through the medium of the conducting layer, and the contributions in these places will be of perfectly normal type, the variations with altitude being determined only by the nature of the variation of conductivity with altitude, and in such a manner as to keep the vertical conduction current density independent of altitude.

duction, would, moreover, cause a continual conduction of electricity between atmosphere and earth, so that a steady state would be reached when the amount of neutralization of charge by this latter process just balanced the charging effect due to the influx of the corpuscles. This theory requires that we suppose the existence of corpuscles of penetrating power so great that they could pass through the whole of the earth's atmosphere, which is equivalent in absorbing power to a column of mercury about 76 cm. high. The greatest range which has been observed in air, for the beta rays of radium, is about seven or eight metres. Electrons having a velocity of 99 per cent. of that of light can travel through only 1.3 cm. of aluminum, which is equivalent in absorbing power to about ten metres of air at atmospheric pressure.

TABLE I.

Showing Variation of Average Range with Velocity.

Velocity of corpuscle / Velocity of light	Average Range Metres.
0.80	0.7
0.85	1.1
0.90	1.9
0.95	3.5
0.99	10.5
0.996	18.0
0.998	26.0

Although, according to electromagnetic theory, the velocity of light represents the maximum velocity which a corpuscle can attain, one must guard against the supposition that, because corpuscles with velocity 99 per cent. of that of light have ranges of only ten metres in air, no corpuscles can have ranges much greater than this. For, electromagnetic theory shows that corpuscles with velocity even 99 per cent. of that of light are very far removed in their properties from those which approach that limit much more closely. As a matter of fact, the mass of a corpuscle increases with its velocity in such a way that the corpuscle must have infinite energy in order to attain the velocity of light. A theory of the absorption of electrons in matter, for high velocities, has been worked out by N. Bohr, and Table I gives, for different velocities approaching that of light, values of the ranges calculated on the basis of Bohr's formula, taken in conjunction with the experi-

mental values obtained by R. W. Varder for the lower velocities up to 0.99 of the velocity of light. On Bohr's theory, the range theoretically attains an infinite value for a velocity equal to that of light. As a matter of fact, for velocities approaching that of light much more nearly than those cited in Table I, additional considerations, which we shall presently discuss, become involved; but, the nature of these considerations is such as to result in ranges even greater than those assigned by Bohr's formula.

A theory such as that of Simpson has interesting consequences in respect of the variation of atmospheric current density with altitude to which it would lead. The ideas in this connection can be most simply studied by considering a case where the whole of corpuscular current through the troposphere is one of negative corpuscles, the positive carriers being caught in the upper atmosphere.

It is of course obvious that, in the steady state, the downward corpuscular current density must equal the upward conduction current density at each point of the atmosphere. Now if the corpuscles are absorbed at all as they descend, the downward corpuscular current density will decrease with approach toward the earth's surface, so that the upward conduction current density will increase with altitude. If, for example, the corpuscular current density should diminish from a certain value at 10 kilometres to half that value at 5 kilometres altitude, the conduction current density would have to increase by 100 per cent. over an increase of altitude from 5 kilometres to 10 kilometres. Our data on the variation of the vertical conduction current density with altitude are not very copious; but, as far as they go, they indicate no very great change with altitude. We thus see that the degree of penetration required by such a theory as that of Simpson is greater than would be necessary to merely account for the corpuscles reaching the earth. It must also account for their reaching with an absence of absorption compatible with the constancy of the conduction current density with altitude.[4]

[4] In so far as the positive corpuscles are absorbed more readily than the negative corpuscles, the inclusion of the former in the argument tends to reduce the rate of diminution of the net corpuscular current density as we descend into the atmosphere from above. Only under very special circumstances would the laws of absorption of the two kinds of corpuscles be such as to make the net corpuscular current density independent of altitude, however.

One might feel more satisfied if he could devise a theory which did not involve such a high degree of penetration as that required by Simpson's theory. There are one or two ways in which we can do this. Suppose we imagine that, owing to radioactivity or some other cause, a certain number of molecules of the air break up per second with the emission of a high-speed negative corpuscle. Then, although these corpuscles may be emitted, on the average, in all directions, the earth will receive some corpuscles from the atmosphere, and it will consequently charge up until the conduction current back to the atmosphere balances the corpuscular current. Practically all of the corpuscles received by the earth will come from a distance comparable with the average range of a corpuscle. We can secure the necessary replenishment with as short a corpuscular range as we wish, provided that we are willing to assume a sufficiently large emission of these corpuscles from each cubic centimetre. Unless we assume a fairly great range, however, we shall encounter difficulties in respect of the variation of the conduction current with altitude. Thus suppose that the average range were 10 metres, so that practically all the corpuscles which came to the earth came from a layer comparable in thickness with this amount. At an altitude comparable with 10 metres the resultant corpuscular current would be zero, because as many corpuscles would be shot upwards from the air below as were shot downwards from the air above. The corpuscular current would, in fact, increase from a maximum at the surface of the earth to zero at this altitude, as would also the conduction current and the potential gradient.

We thus see that if we were to have an equal emission of corpuscles from each cubic centimetre of the atmosphere, and an equal reduction of range in each centimetre of path, it would be practically necessary to assume an average total range equal to the altitude to which we wished to account for a potential gradient. The same result follows if we postulate a definite emission from each *gram* of the atmosphere, for then, although the pressure and density decrease as we ascend, the range increases in like proportion. We can escape from this unwelcome conclusion by postulating an increase of emission per gram with altitude, such as we should have if we were to imagine the emission brought about by some external radiation which was gradually absorbed as it descended. In fact, in order to obtain a constant conduction cur-

rent density, it would be necessary to postulate an increase of emission with altitude, comparable with twice the required corpuscular current, within a weight of air corresponding to the average range of the corpuscles.

It may be of interest to consider the orders of magnitude involved in a theory of this kind. It turns out that if the range is assumed to be 5 kilometres, it is only necessary to suppose that about one corpuscle is emitted per second from every 200 c.c. Now we know that about 6 ions are produced per c.c. per second, so that it is only necessary to suppose that one out of 1200 of these ions is produced with the ejection of a high-range corpuscle.

We may avoid the assumption of a spontaneous emission, and provide for certain other advantages by adopting a hypothesis which bears some resemblance to the foregoing. It has been customary to suppose that our atmosphere is traversed by a very penetrating radiation coming either from the outer regions of the atmosphere, or from some cosmical source such as the sun. This supposition has been invoked to account for the production of ions which is known to occur in a closed vessel freed as far as possible from radioactive air, even after due allowance is made for such gamma radiation from the soil as may penetrate the walls of the vessel. For reasons, the details of which I need not enter into, it has been supposed that this radiation is a form of gamma radiation, but of a type much more penetrating than the radiation ordinarily observed from radium. Now gamma rays possess the power of ionizing, *i.e.*, of ejecting electrons from a gas through which they pass, and the nature of their action is such that the ejected electron is sent out almost entirely in the direction of the gamma rays.

We may thus expect that such a radiation coming from above will eject electrons from the air, and these will travel certain distances in a downward direction before coming to rest. Those electrons which are shot out within striking distance of the earth will reach it and charge it. Their places will be taken by other electrons, which have been shot out from layers above, and become absorbed before reaching the earth. One advantage possessed by this type of corpuscular theory is that it invokes, for the production of the corpuscles, an agency which is already recognized for other reasons, and another advantage lies in the fact that no artificial adjustments of the theory are necessary in order to pro-

vide for a conduction current which is practically independent of altitude. With increase of altitude, the emission of corpuscles per c.c. will be reduced, but the distances from which corpuscles come will be correspondingly increased, so that if the intensity of the penetrating radiation itself remains independent of altitude, the corpuscular current will also remain independent of altitude.

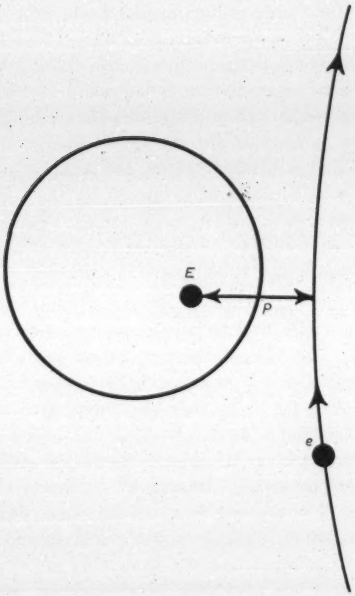
Further, on submitting the theory to calculation, we arrive at magnitudes for the quantities involved which are by no means unreasonable. Thus, if we assume that only 3 high-speed corpuscles are emitted per c.c. per second, a number comparable with that which the penetrating radiation is supposed to eject, it is only necessary to assume that these corpuscles have a range of 9 metres in air in order to account for the replenishment of the earth's charge.[5] We shall presently see that, for reasons concerned with the ionization which would be produced by the corpuscles, it is desirable to endow them with a range greater than 9 metres; but, for the mere requirements of the replenishment of the earth's charge, an average range of 9 metres is sufficient.

Objections to Corpuscular Theories.—I now turn to the two main objections which may be raised against all forms of corpuscular theories. The first of these comes from failure to detect any charging effect on an insulated body exposed to the corpuscles. If corpuscles are being shot into the earth from above, an insulated mass of metal should gradually acquire a charge from the corpuscles which enter it, unless, indeed, the corpuscles are so penetrating as to pass right through it. A fairly thick mass of metal is necessary if the test is to be of value, since corpuscles which have passed through the earth's atmosphere, the equivalent of 76 cm. of mercury, will not show very much absorption in passing through a few additional centimetres of metal. I tried an experiment of this kind in 1915, but failed to find any charging effect of the type sought. More recently, von Schweidler has sought such a charging effect without success. I have always felt that my own experiments, which were done with a copper bar about 25 centimetres long and 5 centimetres in diameter, placed

[5] A theory identical with that here given was developed, quite independently, by von Schweidler, somewhat later than the writer's original publication.

vertically, should be repeated with a larger mass of metal, and Schweidler's experiments were done with a still smaller mass of metal. However, if we should find that a sphere of metal, let us say a metre in diameter, would experience no charging effect, any theory which attributes the replenishment of the charge to influx of corpuscles would be faced with serious difficulty.

FIG. 3.



The difficulty is not insurmountable, however, if one adopts the last of the views to which I have referred, in which the ejection of the corpuscles from the molecules of air is brought about by exceptionally hard gamma rays from above. For, on this view, if the *gamma rays* are sufficiently penetrating to pass right through the metal, they will eject corpuscles from the bottom of the mass as well as inject them at the top. A simple calculation

shows that, provided the intensity of the gamma radiation does not alter in passing through the metal, all that is necessary in order to conclude that as many electrons would be shot out of the bottom of the mass as were shot in at the top, is the assumption that the ratio of the numbers of corpuscles shot out per c.c. of air and metal is equal to the ratio of the densities of these substances, and that the average ranges of a corpuscle, in air, and in the metal, are in the inverse ratio of the densities. Both assumptions are entirely consistent with our knowledge of the laws pertaining to the action of gamma rays and the passage of corpuscles through matter.

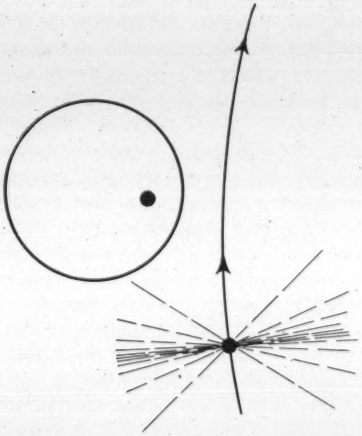
The second great objection, and perhaps the most serious objection, at first sight, to any corpuscular theory, is the fact that we might expect the passage of high-speed corpuscles through the atmosphere, on their way to the earth, to be accompanied by a much greater ionization than is observed. The situation is this: The corpuscular current necessary to balance the atmospheric electric current amounts to an influx of 1500 corpuscles per square centimetre per second. Now we know that an electron approximating in velocity to that of light produces about 40 ions per centimetre of its path, so that, in each cubic centimetre, we might expect ions to be produced to the extent of about 60,000 per c.c. per second, whereas experiment shows that they are only produced to the extent of one-ten-thousandth of this amount. In order to see how we may escape this difficulty, it may be of interest to probe a little more closely the mechanism of the ionization.

Absence of Ionization by Corpuscles with Velocities Approximating That of Light.—Suppose E, Fig. 3, is an electron in an atom, and that another electron e, which I shall distinguish by calling it a corpuscle, approaches the atom. The corpuscle will start to repel the electron as it approaches, and will continue to do so as it recedes, with the result that the electron receives energy, the momentum which it acquires being more or less in a direction perpendicular to the line of flight of the corpuscle. The greater the velocity of the corpuscle, the shorter the time during which the electron has opportunity to receive momentum from it. The efficiency of the corpuscle as regards its power to hurl the electron out of the atom thus diminishes with increase of its velocity, and would, as a matter of fact, become zero if the corpuscle could attain an infinite velocity. The velocity of the corpuscle cannot

attain a value greater than that of light, however, and, as regards the above effect, there is not very much reduction in ionizing efficiency for an increase of velocity from, say, 95 per cent. of that of light, where the ionization has been measured, to the velocity of light itself.

As the velocity of light is approached, however, another phenomenon comes in. The field of the corpuscle does not remain uniformly distributed. According to known electromagnetic laws,

FIG. 4.



its lines of force close up more and more into its equatorial plane, as indicated by the broken lines in Fig. 4. The time which the corpuscle has for acting effectively on the electron is therefore reduced still further on this account; but, the intensity of the action during that time is increased; and, it turns out, as has been shown by Bohr, that, if we take nothing else into consideration, the energy communicated to the electron by the passage of the corpuscle will be unaffected by this concentration of the lines of force.

There is one other very important consideration which we must take into account, however. If an electron receives even a small velocity in a very short time it is known that it will radiate

a large amount of energy. Its sudden start results in a violent jerk in the æther. On submitting this matter to calculation it turns out that, even if we should wish to give to an electron but a small amount of energy, in an infinitesimal time, it would be necessary to pay a sort of tax, of an infinite amount of energy, in the shape of radiation. Now the more nearly the corpuscle approaches the velocity of light, the more suddenly does it communicate to the electron such energy as it imparts. Without entering too greatly into details, we may describe the situation as follows: In the case of a corpuscle moving with a velocity approximating that of light (say 95 per cent. of the velocity of light), but not so closely as to involve these radiation considerations, it turns out that the corpuscle must approach an electron of an oxygen atom within 0.7×10^{-10} cm., in order that it shall be able to eject that electron from the atom. Now it is possible to assign to the corpuscle a velocity so nearly equal to that of light that, if it should pass the electron at 0.7×10^{-10} cm., the radiation tax alone, accompanying ejection of the electron, would amount to more energy than could be imparted by the corpuscle; and, the conditions are such that the situation is still more unfavorable for ejecting the electron if the corpuscle comes nearer to it than 0.7×10^{-10} cm. It is quite true that, in the absence of the radiation forces, the electron would receive more energy the nearer the distance of approach, but with radiation taken into account, the electron when approached within 0.7×10^{-10} cm. by a corpuscle of sufficiently high speed is in the position of a man whose salary is so large that if he took it, his income tax would be greater than his salary, on account of the very large tax on high salaries, so that he would get more if he had a smaller salary. The electromagnetic equations are kind to the electron, however, and allow it to refuse its salary under these circumstances. The net result of all this is that, if the velocity of the corpuscle is sufficiently high, it will be unable to eject the electron if it passes at a distance of 0.7×10^{-6} cm. It will be unable to eject it if it approaches nearer than this, on account of the radiation reaction. But, it would be unable to eject it on passing at a distance greater than this if the radiation considerations were absent. The result is that it cannot eject the electron at all. In submitting the matter to calculation, we find that a corpuscle with a velocity 200 metres per second

less than that of light would certainly be unable to eject an electron from an atom of nitrogen, or oxygen.[6]

It may be of interest to observe that, if we are to suppose that corpuscles come to the earth from outside the atmosphere then, quite apart from all question of absorption in the atmosphere, it is necessary to assume a certain minimum velocity in order to account for their getting here, at least in the vicinity of the equator. For the path of a moving corpuscle is bent by a magnetic field, and it may be bent so much as to cause the corpuscle's return into space. The more nearly a corpuscle approaches the velocity of light, the less is its path bent. The diminution of bending is not so much a direct consequence of the greater speed as it is of the greater mass which results from greater speed. It turns out that, in order that a corpuscle injected into our atmosphere in the vicinity of the equator should be able to reach the surface of the earth without being turned back by the earth's magnetic field, it would have to possess a velocity which is too great to permit it to ionize the air through which it passes.

Birkeland has endeavored to account for the main features of the aurora by supposing it to be produced by the entry into our

[6] Without invoking these considerations, and utilizing only the older views, von Schweidler (*Akad. Wiss. Wien Ber.*, 1918, 127, 2a, pp. 515–533) has concluded that the ionization per centimetre of path should diminish indefinitely as the corpuscle approaches the velocity of light. It would appear, however, that this argument cannot be substantiated. It is based on the fact that the older theories give the ionization per centimetre of as inversely proportional to mV^2, where m is the mass, and V the velocity of the corpuscle. For slow velocities, $\frac{1}{2}mV^2$ is the kinetic energy, so that the statement is made that the ionization per centimetre of path is inversely proportional to the kinetic energy. Von Schweidler, taking this statement, substitutes for $\frac{1}{2}mV^2$, the relativity expression for the kinetic energy, which becomes infinite when V approaches the velocity of light, and thus concludes that the ionization per centimetre of path should diminish indefinitely as that limit is attained. It is to be observed, however, that $\frac{1}{2}mV^2$ occurs in the original expression for the ionization per centimetre of path not because it is the kinetic energy. The occurrence of V^2 in the denominator of that expression is closely bound up with the fact that the time during which the corpuscle is in effective proximity to the atom diminishes with increase of velocity. In fact, this quantity V^2 is not the representative of something which should really become infinite when V attains the velocity of light, as a detailed examination of the elements involved in the calculation of the ionization per centimetre of path will clearly show.

atmosphere of high-speed corpuscles from the sun. In order to account for the facts as regards the bending which the paths of these corpuscles should suffer in the earth's magnetic field, he is forced to assume velocities ranging from 400 metres per second less than the velocity of light to 4 metres per second less than that limit, so that the velocity at which ionization should cease falls well within the limits assigned by Birkeland. It is quite true that the present theory leads to the conclusion that Birkeland's corpuscles would not ionize at all, so that they could not well account for the aurora; but, it is now generally believed that the aurora is not caused by the ionization produced by negative electrons, and I only cite this connection with Birkeland's calculations to show that, in invoking speeds only 200 metres per second less than that of light, we are not doing anything more drastic than has been done already in other branches of the subject.

It may be remarked, moreover, that if we extrapolate Bohr's theory of absorption to velocities comparable with 200 metres per second less than that of light, we find that corpuscles having this velocity could pass through distances in the atmosphere comparable with a kilometre. When we recall that transfer of energy to electrons in the atoms constitutes the process by which the corpuscles lose energy on Bohr's theory, we see that the reduction in this transfer, symbolized by failure to ionize, will still further increase the possible range. Thus, the nature of all the considerations involved is such as to result in mutual consistency in the views that if corpuscles should enter our atmosphere with speeds sufficiently near that of light to enable them to reach the earth's surface in the vicinity of the equator, without being turned back by the earth's magnetic field, they would pass right through the atmosphere with limited absorption, and would fail to ionize the air through which they passed.

It would thus appear that the difficulties attending the ionization which might be expected to follow, at first sight, from any corpuscular theory of the earth's charge are not unsurmountable. But while, as regards the mere replenishment of the earth's charge, we could avoid the assumption of long ranges, as in the theory which invokes the penetrating radiation to eject the corpuscles from the air, we have to postulate, for the corpuscles, velocities

closely approximating that of light; in order to explain the absence of ionization, and this in itself implies long range as a consequence.

Attempts have been made to overcome the supposed difficulty attending the absence of any charging effect in the case of an insulated body by supposing the influx of corpuscles to take place in the auroral belt. It is perhaps worth while pointing out that, even should we do this, we could not escape the difficulty attending the ionization by these particles, unless we adopt some such theory as I have sketched; for, if we assume the ordinary ionization per centimetre of path, it becomes necessary to invoke recombination, to cut down the resulting ionic density, to such an extent that we must suppose the area of precipitation to be less than one-thousandth of the area of the earth, as otherwise the conductivity produced in the air would be so great that, for a potential gradient of 150 volts per metre, we should calculate, for this region alone, a total conduction current greater than the corpuscular current.

THE ORIGIN OF THE EARTH'S MAGNETISM.

The fact that the earth possesses the power of controlling the direction in which a magnetic needle points was known to the ancients. The earth acts, in fact, approximately as though it were a huge magnet with two poles, one in the vicinity of the north geographical pole, and the other in the vicinity of the south geographical pole. The problem of accounting for this condition of affairs has been one of the most fascinating fields for speculation in cosmical physics.

Of course we may say that the interior of the earth contains, in a large degree, a magnetizable material like iron, and that it got magnetized somehow or other, at some time or other, and that is the end of it. We cannot shuffle off responsibility for an explanation quite so easily, however; for, we know that if we heat a steel magnet to a temperature of 785 degrees centigrade it loses its magnetism, and we have reason to believe that temperatures considerably higher than this are attained within the earth. We can escape this conclusion provided that we are willing to suppose that the great pressures inside the earth raise the temperature at which magnetism is lost; but, until further evidence upon the effect of pressure in this respect is available, one is tempted to seek some

more fundamental cause as responsible for the earth's magnetic state.

It is hard to resist the temptation, which has been felt by many, to suppose that the earth's magnetism is bound up in some way with its rotation. It is true that most simple theories based on this view lead to a state of magnetization symmetrical with the geographic axis, and do not, moreover, spontaneously suggest that remarkable characteristic of the earth's magnetism exemplified in its secular variation. Nevertheless, one might feel well satisfied with his progress if he could see any reasonable way in which a body like the earth could acquire magnetization of any kind comparable in amount with the earth's magnetization.

Theories Involving Rotation of Electrostatically Charged Systems.—We know that an electric current flowing in a circular wire gives rise to a magnetic field; and, if we were to wind the surface of a sphere with a single layer of coils, and then pass a current through the coils, we should obtain a magnetic field approximating in type to the earth's field. Now since the earth is electrically charged, the first idea which might occur to us is to see whether the current formed by the rotation of this charge with the earth will give an appreciable field. The charge density is known, and the velocity of the earth's rotation is known, so that it is a comparatively easy matter to compute the effect. Neglecting the effect of the positive charge in the atmosphere, we find that the magnetic field produced amounts to only the one-hundred-millionth part of the earth's field. In so far as the earth's charge produces a very considerable electric field at its surface, we see that a charge density necessary to give a magnetic field comparable with the earth's field would give rise to an electric field almost inconceivably large. Moreover, it is well known that, on account of the effect of the motion of the observer with the earth's surface, the field would appear to him different from the field which would be observed by a stationary observer, and the nature of this effect of the observer's motion is such as to cause the vertical component of the field to have the wrong sign if the sign of the charge is adjusted to give the correct sign for the horizontal component.

When we take the motion of the atmospheric charge into account, the result obtained is still more curious. To a stationary

observer, the vertical component is now zero everywhere,[7] while to an observer participating in the earth's motion, both the vertical and the horizontal components are zero.

There is, however, another way in which we may obtain a magnetic field by the rotation of charges, without the necessity of imagining any external electric field. If we imagine a sphere of positive electricity and set it in rotation, it will give rise to a magnetic field; but, for a given total charge, the strength of the equivalent magnet will be greater the greater the diameter of the sphere. Thus, if we superpose two spheres of slightly different sizes, one sphere being made of positive, and the other of negative electricity, but each containing the same total charge, they will act as oppositely directed magnets of different strengths, and there will be a residual magnetic effect. As far as the electric field is concerned, however, there will be no residual effect at points external to the sphere. Now, modern physics tends to the belief that all matter is composed of nothing but positive and negative electricity; and, it appears that, while a cubic centimetre of, say, iron is neutral as a whole, nevertheless, the amounts of positive and negative electricity in it are so great that if we could separate them and concentrate them in two points one centimetre apart, they would attract each other with a force equivalent to 10^{20} tons. In view of this very large amount of the electricity of each sign present, it turns out that we may account for a magnetic field comparable with the earth's field, if we are willing to suppose that the positive electricity in the earth is distributed throughout a sphere which is less in radius then the sphere of negative by only 2×10^{-8} cm.,

[7] At first sight, this statement seems to be inconsistent with the fact that the tubes of magnetic induction from closed curves, since they are apparently prevented from escaping through the positive layer, or into the earth, on account of the absence of a vertical component. The point is that, owing to the thinness of the positive layer, the volume of the space between the earth's surface and a sphere within which practically all the positive charge lies is relatively small. The total magnetic flux across any annular section of this space is small, so that a practically infinitesimal vertical component is all that is necessary to permit of the entry and exit of the lines of magnetic induction to and from the space, since the area available for such entry and exit is the whole area of the earth's surface. Indeed, that there is nothing fundamentally mysterious in the nature of the field here cited will be recognized when it is pointed out that it is simply the field obtained by superposing on the field due to a uniformly magnetized sphere, a uniform field parallel to the axis of magnetization.

i.e., we need only imagine the diameters of the two spheres to differ by as much as that of a single molecule.

However, in spite of the very strong temptation to lean to this view, which was first considered by Sutherland, we are confronted with a very serious difficulty when we come to seek an explanation of why the positive and negative can remain separated by even this small amount; for calculation shows that, at the surface of transition, there would be an electric field of one thousand million volts per centimetre protesting against this separating of the charges.

There are a number of causes which would operate in some degree towards producing separation of charges such as would give rise to a magnetic field as a result of the earth's rotation. Thus, it is customary to suppose that, in a solid, a large proportion of the electrons are not bound in the atoms but are free to roam about in the interstices between the molecules in very much the same way as the molecules of air fly about between the leaves of a tree. Owing to the centrifugal force of the earth's rotation, these electrons will tend to be thrown outwards. This tendency will be resisted by the attraction of the positive electricity which they would leave behind. Still there will be a small tendency in this direction; and, as a result of the rotation of this system in which the charges have been displaced, we should obtain a magnetic field. However, on submitting the matter to calculation, it turns out that the field so obtainable amounts to only 10^{-23} of the earth's magnetic field, and is, moreover, of a type differing widely from it.

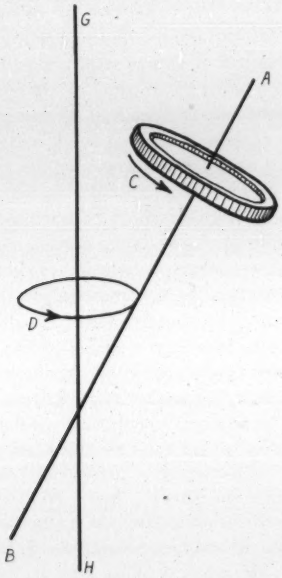
Again, owing to the earth's gravitation, the electrons will tend to move towards the centre. Apart from sign, this leads to the type of displacement postulated in Sutherland's theory; but, when we work out the magnetic field, we find that it amounts to only 10^{-21} of the earth's field, and is, moreover, in the wrong direction.

I may refer to still another possibility. We know that the inside of the earth is hotter than the outside. Now if we have a temperature gradient in a gas, the gas will become less dense where the temperature is high than where it is low, so the electrons will tend to become less dense in the hot regions than in the colder, this tendency being resisted by the electrostatic attraction which is set up as a result of the separation. On working this case out, we find that the magnetic field resulting from it amounts to

only 10^{-17} of the earth's field, although it happens to be in the right direction.

There are a number of other possibilities somewhat similar to those which I have discussed; but, on submitting them to calculation, they all lead to fields of an order of magnitude quite infinitesimal compared with the earth's field. I think we may con-

FIG. 5.



clude, as a general rule, that it is practically hopeless to seek an explanation of the earth's magnetism on the basis of the rotation of charges which have been separated against electrostatic attraction, since the mechanical forces necessary to produce the required separation must be, in all cases, enormous.

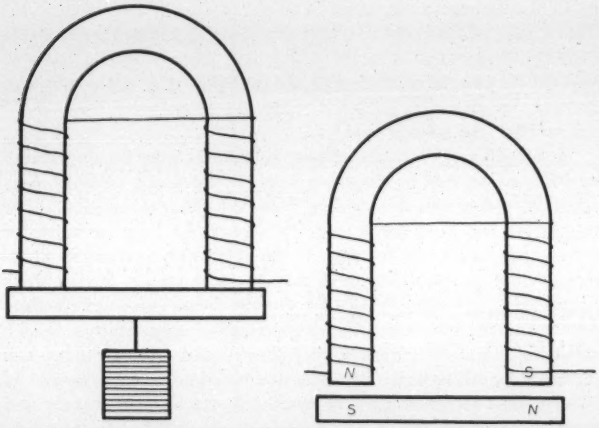
Theories Involving Gyroscopic Action.—Let me now turn to another possibility. We know that the molecules of iron tend to act like little magnets, and the magnetization of a steel bar consists in the process of partially orienting these little magnets so

as to point more or less in one direction. Now a circular hoop of wire carrying a current acts like a magnet with its axis perpendicular to the plane of the hoop; and it is generally supposed that an atom receives its magnetic character on account of the circulation of electrons within it. This circulation of electrons makes the atom act like a gyroscope. Now it is well known that, if a gyroscope be set in rotation about its axis *AB*, Fig. 5, in the direction indicated by the arrow *C*, for example, and if this axis is then caused to rotate about another axis *GH* in the direction indicated by the arrow *D*, it will tend to turn parallel to *GH*. If, therefore, the earth is composed largely of iron, there will be a tendency for its molecular magnets to turn their axes parallel to the axis of rotation. That a piece of iron may be magnetized by rotation has been experimentally demonstrated by S. J. Barnett; but, when we come to apply these results to the earth, we find that it is not possible to account, by this means, for a magnetic field larger than about 2×10^{-10} of the earth's field.

It is well known that different substances may be magnetized to different degrees by the same magnetizing force. Easily magnetizable substances are said to have a high permeability. The suggestion has been made that, on account of high pressure, or some other cause, the interior of the earth may possess an enormously high permeability, and that, in consequence, a very weak magnetizing force, such as that arising from gyroscopic action, may be sufficient to endow it with appreciable magnetization. Such a view must, however, meet a very serious objection, which I can illustrate by calling your attention to a well-known experiment. If a horse-shoe electromagnet be excited by an electric current and caused to hold up, say, a 100-pound weight attached to its armature, it will be found that the weight will continue to be held after the current is stopped, provided that the pole faces, and the armature, are truly plane. The complete iron circuit composed of horse-shoe and armature having been magnetically excited, remains so after the stoppage of the current. If now the armature be forcibly removed from the horse-shoe, magnetic poles appear at the points of separation, as indicated in Fig. 6. The action of these magnetic poles is such as to destroy the state of magnetization previously existing in the horse-shoe and armature, and, as a result, it is found that now, not only will the horse-shoe refuse to support the armature and 100-pound weight, but it will even refuse to support

the armature alone. It is much more difficult to magnetize a short piece of iron than a long one, on account of the proximity of the poles to the main body of the iron in the former case. In fact, these poles produce, within the iron, a magnetic field which is in such a direction as to tend to demagnetize it. In the case of a magnetized sphere, the effect of the poles is to produce within the body of the iron, a demagnetizing field equal to the field which the sphere itself produces at an external point on its equator. Hence, any primary magnetizing agent which is to be ultimately responsi-

FIG. 6.



ble for the production of the earth's field by orientation of the molecular magnets, must be of such intensity that it will produce, on the molecular magnets, forces at least equal to the forces which would be produced on them by a magnetic field equal in intensity to the earth's magnetic field at the equator.

Theories Involving the Circulation of Currents Within the Earth.—The earth's field could be accounted for if we could account for a suitable circulation of currents in the earth's interior. Such an attempt at explanation immediately calls, however, for an explanation of the electromotive forces wherewith to sustain the currents. In this connection, it is not without interest to recall

a calculation made by H. Lamb, many years ago, the significance of which I shall endeavor to explain.

If we start a current in a wire, and then leave it to itself, it will die to zero almost immediately on account of the resistance offered by the wire. The situation is analogous to what we should have were we to take a hollow anchor ring, fill it with water, and then give it a sudden rotation. If there were no friction, the water would go on swirling around for ever; but, on account of the friction, it is soon brought to rest. If the water were less dense, and had therefore less inertia, it would be brought to rest sooner. The analogy between the motion of the water and that of electricity is not very close, but the electric current does possess the property of inertia, and it turns out that the inertia of a current is greater in a large hoop than in a small one. So important does this inertia become when dealing with a body of the size of the earth that, as Lamb has shown, if a system of currents were started in a sphere of the earth's size, having a conductivity equal to that of copper, and if the electromotive forces which were responsible for starting them were removed, it would take about ten million years for the currents to decay to one-third of their original values. Attempts to account for the earth's field in this way have met criticism on the basis of the enormous currents which would be calculated by extrapolating back, even to epochs not more remote than those during which the earth's crust has been solid, so that, unless there is some reason for supposing that the conductivity is, or has been in the past, even greater than that of copper, we are confronted with the necessity of accounting for enormous amounts of energy necessary to have produced the magnetic field originally.

The actual current density within the earth, necessary to account for the earth's field, is very small, being, for example, only the one-hundred-millionth of an ampere per square centimetre on the surface at the equator, for the case where the current density is proportional to the distance from the axis of rotation. I have already referred to the largeness of the amounts of positive and negative electricity in a cubic centimetre of the earth. If, taking a sphere of iron of the size of the earth, we assume that there are about 10^{23} free electrons per cubic centimetre, a not unreasonable number, we find that it is only necessary to suppose that the electrons at the earth's surface differ in velocity from the matter in which they reside by one part in 7×10^{16} of the velocity of that

matter in order to account for a current of one-hundred-millionth of an ampere. It is perhaps not too much to hope that a fuller knowledge of the mechanism of conduction of electricity in solids than we have at present may lead to an explanation of such a small difference as arising directly on account of the earth's rotation. It may not be a question of accounting for an electromotive force wherewith to overcome the electrical resistance of the material of the earth, but rather of showing that, in a rotating body, the condition of electronic motion which is permanent, the motion which must exist in order that there shall be no further degradation of energy into heat, is one in which there is a finite relative motion between the free electrons and the substance of the rotating body.

A small relative velocity between the free electrons and the substance of the earth may be accounted for on ordinary lines if we assume that the velocity of the earth is diminishing on account of tidal action. If the retarding forces operated on the earth and not on the free electrons the latter would tend to retain their angular velocity undiminished. As a matter of fact, they would actually lag behind the remainder of the matter to such an extent that the force brought into play upon them, on account of this lag, a force calculable in terms of the specific electrical resistance, was just sufficient to cause them to follow the earth in its retardation. However, on submitting this matter to calculation, it appears that the earth would have to be retarded at such a rate as to lose the whole of its velocity in less than one day if the current thus produced were to be sufficient to account for the earth's field.

We can account for a small magnetic field if we suppose that, while the negative electrons participate in the translatory motion of the matter in which they are contained, they do not rotate with reference to axes fixed in space. The effect in such a case is, in fact, to give rise to a magnetic field which is the net equivalent of what we should have if everything were stationary except that each electron rotated in a direction opposite to that of the earth, about an axis in itself, once per day. The magnetic field produced in this case amounts to only 1.6×10^{-25} of the earth's field and is moreover in the wrong direction. It may be of interest to observe, however, as a mere matter of numerical interest, without any implication of a bearing on the earth's magnetism, that if we could divide all of the positive electricity in the earth into groups about a centimetre in average radius, and then see some way by

which these little groups could go around with the earth without rotating, while all the negative electricity participated in the rotation as well as the translation, we should have a way of accounting for a magnetic field of the order of magnitude of the earth's field.

Possibilities Inherent in a Slight Modification of the Fundamental Laws of Electrodynamics.—There is always the chance that the origin of the earth's magnetism may have to be sought in some fundamental but small departure from the ordinary laws of electrodynamics. In this connection, I would remind you of a somewhat analogous modification which was suggested by H. A. Lorentz, to account for gravitation. Lorentz showed that if we should imagine that the attraction between a positive and a negative electron exceeded the repulsion between two positive or two negative electrons, by one part in 3×10^{36}, the residual attraction would be sufficient to account for gravitation. After paying all due regard to the care necessary in defining what we mean by a neutral body in such a case as this, Lorentz's theory may be shown to lead to the conclusion that, in order that the free electrons in a body may be in equilibrium, the body must acquire a charge density to an extent not wholly determined by the weight of the electrons. Schuster has discussed the possibilities in this connection; but it would appear that, under the most favorable assumptions, the density would be insignificant as regards producing, by its rotation, a magnetic field comparable with the earth's field.

A rather greater measure of success is attained by making a somewhat similar assumption with regard to the magnetic field produced by a moving charge. I will remind you of the fact that, when a charge is in motion in a magnetic field, it experiences a force in a direction perpendicular to the plane containing the direction of its velocity and the direction of the magnetic field. It is of course, this force which is responsible for the operation of the electric motor. Even when we measure a magnetic field in terms of its action upon a magnet, it is this force which is really at the basis of the action, since the magnet derives its properties from the electrons revolving within it. In analogy with the case of electrostatics, where we have to deal with the forces produced by positive on positive, negative on negative, and positive on negative, we have in addition, for moving electrons, the force due to the motion of a positive electron on a moving positive electron, the

force due to the motion of a moving negative electron on a moving negative electron, the force due to the motion of a negative electron on a moving positive electron, and the force due to the motion of a positive electron on a moving negative electron.[8] If, for similar motions, these four forces are all equal, a moving electron, or a magnet, would be entirely unaffected by the rotation of the earth as a whole. If, however, the forces, due to motion, between unlike moving charges are suitably different from those between like charges in the same states of motion, it will immediately appear that the electrically neutral earth will, by its rotation, produce those forces on magnets and moving electrons which we associate with a magnet as ordinarily defined. By making the forces between electrons of like sign equal for both signs, the force due to the motion of a negative electron on a moving positive electron greater than, and the force due to the motion of a positive electron on a moving negative electron less than the forces between like electrons to the extent of about two parts in 10^{16}, we can account for the equivalent of a magnetic field of the order of magnitude of the earth's magnetic field.[9] If we wish to combine these alterations with suitable alterations in the electrostatic forces, we can also include gravitation in the complete scheme.

The Bearing of the Secular Variation.—There is some evidence to support the belief that the earth's magnetic axis rotates about the geographic axis once in about 500 years. Now we know that if, by any means, we cause the number of lines of magnetic induction passing through any area of a conductor to change, induced currents will be set up in the conductor. This is merely another aspect of the inertia associated with electromagnetic phenomena, to which I have already referred. The movement of the earth's magnetic axis within the earth itself will therefore give rise to induced currents, and the magnetic field which we observe will be that due to these induced currents (the secondary field), and that due to the primary causes (the primary field). Taking an iron sphere of the size of the earth for purposes of illustration, it turns out that the flux of magnetic lines of induction of the secondary field through

[8] The first two of these may serve as the means of defining the magnetic fields due to motions of positive and negative electricity respectively since, on the present lines, each of these needs independent definition.

[9] Incidentally it may be remarked that it is possible to make the modification of the law in a form consistent with the restricted theory of relativity.

the sphere, which flux is of course related to the primary field, is of such a magnitude as to almost completely annul that component of the primary flux which is perpendicular to the axis of rotation. As a result, there is left only a small residual non-axial component. Thus, in order that the resultant flux shall have an appreciable inclination to the geographic axis, it is necessary for the primary axis to lie very near to the equatorial plane, and yet for the primary flux to be so large that its axial component which is small compared with it, represents the axial component which we observe. This practical coincidence of the primary axis of magnetization with the equatorial plane would be, of course, very remarkable were it true. The considerations involved in deducing it involve the assumption of an electrical conductivity comparable with that of iron; and, while it is not implied that the main body of the earth has a conductivity of this order of magnitude, it may nevertheless be of interest to call attention to the curious results which would follow from the secular variation if it had.

The Diurnal Variation.—The magnitudes of the earth's magnetic elements go through fairly regular variations throughout the day and throughout the year; and, we are in a much better position for an explanation of these than we are for the explanation of the earth's field as a whole.

When a conductor moves across the lines of force of a magnetic field, currents are induced in it, this being, of course, the principle at the basis of the operation of the dynamo. Now the air is continually in motion under tidal action, just as the sea is in motion; and, as it moves it cuts across the earth's lines of magnetic force. Electromotive forces are consequently produced in the atmosphere; and, these give rise to currents to an extent determined by the conductivity of the atmosphere. The currents result in magnetic fields, and these magnetic fields, being thus related to the atmospheric tides, may therefore be expected to experience diurnal and seasonal variations. Such was the suggestion originally made by Balfour Stewart, and subsequently worked out in detail by Schuster.

It will be observed that the magnitudes of the effects to be expected depend upon the conductivity to be assigned to the atmosphere. The lower atmosphere is so poor a conductor that its effect in contributing anything to the phenomenon is negligibly small. There are, however, reasons for believing that the conductivity of

the upper atmosphere is very much greater; and, for certain reasons into which I need not enter, Schuster has supposed that appreciable conductivity may be considered as confined to a layer of the atmosphere 300 kilometres thick, situated at an altitude where the average pressure is one-millionth of an atmosphere. The conductivity which he finds necessary to account for the phenomena is, however, three hundred thousand million times as great as the conductivity at the earth's surface. To realize what this means it may be remarked that, with such a conductivity, a column of air extending completely round the earth, in the upper atmosphere, would have a resistance no greater than that of a column of equal cross-section and only one-thirtieth of a centimetre long, at the earth's surface. In other words, such an assumption means, to all intents and purposes, that the earth may be supposed to be surrounded by a conducting shell. Startling as this assumption may seem, it falls in well with the requirements of other phenomena. Thus, in discussing the origin of the earth's charge, we had occasion to consider the possibility of the existence of such a layer. Again, it has been for long a matter of contention as to why wireless waves travel such great distances over the earth's surface. On elementary considerations we should suppose that their energy would be dissipated into space, and that very little would succeed in travelling far around the earth. On the other hand, the conducting shell provides us with a way out of the difficulty. For, such a shell would reflect the waves in a manner analogous to that met with in the case of sound waves transmitted round a whispering gallery, or through a speaking tube. Of course the same considerations would apply to show that wireless waves coming from the outside would fail to get inside the conducting shell, for they would be reflected at its surface. We must not apply this argument with too great generality, however, for light is really an electromagnetic disturbance of very short wave-length, a sort of very short wireless wave; and, it would be unfortunate for the theory were it to show that light cannot come through the shell.

It is probable that the ultra-violet light plays no important rôle, as it is capable of accounting for a conductivity less than one-millionth of the conductivity required. Other agencies which are frequently invoked to account for the conductivity are corpuscular radiations from the sun. Some radiation of this kind seems necessary to account for the aurora.

The Aurora.—Auroral displays take varied forms, and are of varied hues and colors, but they are confined mainly to high latitudes. A characteristic feature of the aurora is the sharp boundary at its lower edge. The height of this boundary has been measured by various observers, and appears to vary from 70 to 300 kilometres with a maximum number at about 115 kilometres. Auroral activity is confined largely to two zones of the atmosphere situated at an angle of about 20 degrees to the polar axis. One of the best-known theories of the aurora is that which attributes it to light caused by the ionization of our atmosphere, by negative electrons emitted from the sun. We know, from laboratory experiments, that beams of swiftly moving charged particles are deflected by a magnetic field; and, Birkeland, on the experimental side, and Störmer, on the mathematical side, have done much work with the object of finding out how far the characteristics of the aurora can be accounted for on the view that it is caused by a stream of negatively charged particles which come from the sun, and are precipitated into our atmosphere under the bending action of the earth's magnetic field. The two zones of active precipitation become satisfactorily accounted for, except that they occur at latitudes higher than those corresponding to observation. The cause of the difficulty is the small mass of the electron, which results in too large a bending of the paths by the earth's magnetic field. Now, as I have remarked earlier in this paper, the mass of a corpuscle increases with its velocity, and becomes theoretically infinite for a velocity equal to that of light; but, to such an extent has Birkeland found it necessary to draw upon this fact, that he must suppose the corpuscles responsible for the aurora to have velocities ranging between 400 metres per second less than that of light and 4 metres per second less than that limit, in order to make the theory fit the facts.

In order to overcome the difficulty inherent in the small mass of the electron, Vegard and others have supposed that the particles responsible for the aurora are not electrons, but alpha particles emitted by the sun. Alpha particles would have about the right velocity and mass to provide for the necessary bending. Moreover, the characteristic features of alpha particles are the straightness of their paths, and the very sharp limit to the range of their action. Both of these are well typified in the aurora, the straightness of the path by the straightness of the streamers, and the sharp

limit of the range by the sharp lower boundary of the luminescence. The fact that alpha particles only travel for a distance of a few centimetres in air at atmospheric pressure presents no difficulty, for the pressure in the auroral regions is very small. As a matter of fact, the range of the alpha particle is amply sufficient to account for the depth of penetration; and, in so far as there is any difficulty in this matter at all, it arises from the fact that, on certain plausible views as to the composition of the upper atmosphere, the range of ordinary alpha particles is rather too large to account for the facts.

Not only for the explanation of the aurora has the aid of alpha particles from the sun been invoked, but they have also been called in to account for those sudden perturbations of the earth's magnetic field known as magnetic storms. These perturbations are most frequent at times when the number of sun-spots is a maximum, so it is natural to attribute them to something emitted from the sun-spots.

As illustrating the difficulties confronting any explanation of these storms in terms of alpha particles, I may quote a few of the objections which have been raised by Lindemann. In order to account for the facts, it becomes necessary to suppose the particles to be shot out in a sort of cone. Now Lindemann points out that, in the first place, the magnitudes of the phenomena involved are such as to necessitate an incredibly large amount of radioactive material in the sun; in the second, the alpha particles could not remain together as a beam on their journey from the sun to the earth, on account of self-repulsion. A beam of the necessary intensity would show a lateral spreading corresponding to an acceleration of the order 10^{13} cm./sec.^{2} at its boundary. Finally, even though the alpha particles could reach the earth's atmosphere as a beam, they would charge it at such a rate that, after a few seconds, no more could come here on account of the repulsion of those which had already arrived.

Lindemann himself puts forward the view that the particles which enter the atmosphere from the sun, and are responsible for magnetic storms, and for the aurora, are really gaseous ions shot out from the solar prominences. These prominences, it will be recalled, are composed of flaming masses of gas, some of which attain the incredible heights of 300,000 miles; and their forms change so rapidly as to tell us that the gases which compose

them must move with stupendous velocity. Velocities of the order 80 million centimetres per second are not uncommon.

We know that if light falls upon a particle, a pressure is exerted upon the surface of the particle; and the smaller the particle, the more effective is the light in producing velocity therein. It will be recalled that it is customary to assume that the reason why comets' tails bend away from the sun rather than towards it, as gravitation would lead us to believe, is that the pressure of light tending to drive the particles away from the sun is, on account of their small size, more effective than gravitation which is pulling them toward it. Lindemann has consequently supposed that the huge velocities attained by the gases in the solar prominences are attained under light pressure; and he further shows that, as a consequence, some of this gas may be expected to shoot out into space with such high velocity as to give the atoms properties similar to those of alpha particles. He supposes that the gas is almost completely ionized on account of the high temperature in the prominences, so that the particles shot towards the earth are of both signs, thus securing absence of spreading due to self-repulsion.

I must not enter into the details of Lindemann's theory of the phenomena. I would point out, however, that, from various points of view, we are lead to the conclusion that high-speed corpuscles of some kind or another are shot into our atmosphere from the sun and are probably ultimately responsible for magnetic storms, for the high conductivity of the upper atmosphere, and for auroral phenomena.

It is very fortunate that our knowledge of the upper atmosphere rests practically completely upon conjectures, and that we have no direct experimental data on the electrical conditions at the altitudes concerned. If only we could get up there with some apparatus we should be much better off.

GRAVITATION.

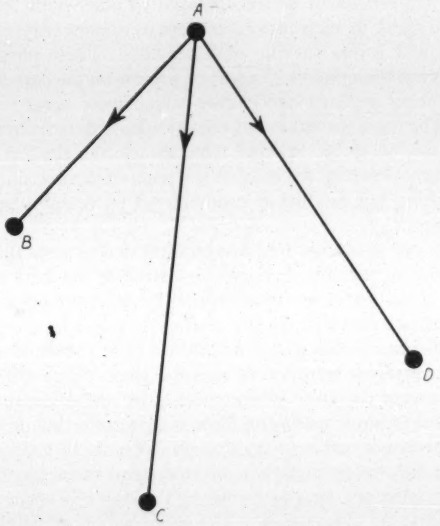
It is not possible to deal adequately, with the great problem of gravitation, in a short space, and I must confine myself to but a few remarks concerning it. In seeking an explanation of such a phenomenon as gravitation it is perhaps not inappropriate to inquire how much we are justified in expecting from nature in the way of a cause. We say, or some of us say, that Newton discovered the force of gravi-

tation, and we have occupied ourselves ever since in trying to find the cause of gravitation. Now what sort of an explanation should we be content with? Well, the picture of the earth whirling around the sun without flying away from it strongly suggests to the mind the picture of a stone whirling around at the end of a piece of elastic, so we are tempted to picture some sort of elastic pull emanating from the sun. Now suppose that somebody had succeeded in accounting for gravitation in terms of the elastic properties of an intervening medium, how much comfort would we be justified in extracting from this? Shall we say, now all is clear, there is in reality not a great nothingness surrounding the sun, but a medium with elastic properties, and this medium does the pulling. If we do, some pestering philosopher may say: but, why does the elastic pull? Well, we shall answer, that is a matter of cohesion. The elastic is composed of a lot of little molecules and when these are separated from each other, they tend to return. But, says the philosopher, why do they do this? We reply, although the molecules are apparently separated from each other there is really a medium between them, and this medium is endowed with elastic properties and does the pulling. But, says the philosopher, why does the elastic pull? And so we may go on discussing the matter *ad infinitum*. We cannot in fact come to any ultimate explanation of the matter. The very best we can do is to conclude that this phenomenon of gravitation for which we cannot account, acts in many respects like that other phenomenon concerned with the elastic for which we also cannot account.

Now in the statement of Newton's law of gravitation there is really no reason to speak of force at all. What Newton found is that if, for example, a particle is moving far away from all other bodies, it moves in a straight line with constant velocity, but if two particles come into each other's vicinity they approach each other with an acceleration inversely proportional to the square of the distance between them. In general, he found that one could describe the motion of a particle A in this way: Join the particle to any other particle B in the universe. Write down, for the particle A, an acceleration in the line joining it to the particle B and proportional to the inverse square of the distance between them, and to two numbers, the masses, characteristic of each. Then draw a line to another particle C, and write down another acceleration, in the line adjoining A and C, and proportional to the inverse

square of the distance, and to two mass numbers characteristic of A and C. Do this for all the particles in the universe. Compound the accelerations by the usual vectorial procedure, and the result will represent the acceleration of the particle A. In this statement of the law as a mere empirical fact, we do not need to use the word force at all. The law is complete without it, and is sufficient to enable us to calculate all the planetary motions. However, the

FIG. 7.



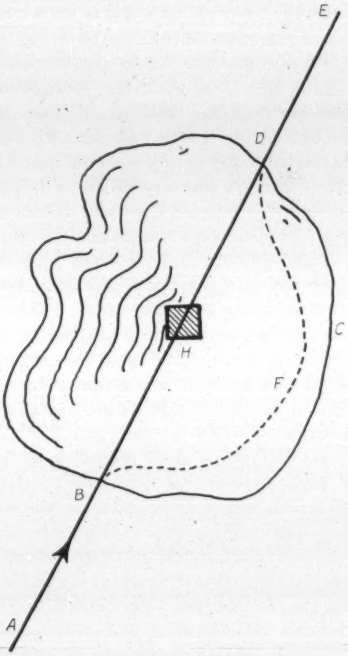
fact that the resultant acceleration is composed of a number of non-interfering parts, each depending on one other particle, and in the line joining A to that particle, strongly suggests to the mind the idea of an elastic pull. The mind likes to think of the elastic or of something; for, it is very difficult for the human mind to think at all unless it has something to think about. If Newton had found that the acceleration contributed by B, for example, was not in the line adjoining it to A; if the acceleration had been at an angle of 45 degrees to this line, it would still have been possible to speak of the force due to B and A; but, the mind would have lost much of

the inspiration which is derived from thinking of the elastic. A little further complication of the situation would result in our receiving still less benefit from the analogy of the elastic. We might hold on for some time, and force ourselves to think of a sort of modified elastic which could act in this way; but in order to do so, we should have to introduce notions more and more bizarre. It is by attempting to force all the laws of physics into the forms with which we were accustomed to deal in the first branch of that subject which we encountered in our youth that we are driven to speak of such curious concept as masses varying with the velocity, and forces varying with velocity. These physical monstrosities are the symbols of a sort of protest on the part of nature's laws at being strained into clothes which were never made to fit them. The more the motion of the particle departs from the simple type of motion to be expected from the elastic, the less the mind gains from picturing the motion in terms of forces, until eventually, nothing but confusion results from an attempt to preserve the analogy.

From our youth up, we have been trained to view things from the standpoint that a body moves in a straight line with a constant velocity if no forces act upon it, and doubtless many of us have a very definite idea that we know what we mean by the statement. Our confidence in this surety might be a little shaken if we should imagine ourselves transported to some place where nothing is to be seen except the body whose motion we are discussing. How are we now to know whether a force is or is not acting on the body? It will surely not suffice to say that no force can be acting on it because we can see no apparatus around us to cause the force; for waves, or what not, may be travelling through this space and causing all sorts of phenomena. The more we think of the matter the more we realize that there is no meaning to the statement of whether a force is or is not acting upon the body unless we make one. In fact, the condition that the body is moving in a straight line with a constant velocity is an embodiment of what we *mean* by absence of force. This is the criterion of nothingness which the mind takes as its origin from which to elaborate its thoughts. The body moves in a straight line with constant velocity, and the mind is content with the spectacle: It is not tempted to do any thinking. If the body does not move in this way, the mind decides that a duty devolves upon it to do some thinking about the matter.

It thinks of the situation as a departure of the motion from one of the uniform rectilinear type, and such circumstances as it finds associated with this type of motion and not associated with the uniform linear motion it regards as causes of the departure. A full appreciation of the degree of arbitrariness of the mind in un-

FIG. 8.



consciously importing fundamental reality into the forces with which it deals is necessary if we are to understand the language of such a law of gravitation as that formulated by Einstein. It so often happens in arguments having to do with the fundamentals of physics, that the secret of understanding a certain viewpoint which at first repels us, because we think it lacks some reality which

we believe we find in some other viewpoint, is a more clear realization of the fact that the reality which we thought we saw was itself but a figment of the imagination.

I cannot of course enter, at this time, into the details of Einstein's theory of gravitation; but, I can possibly indicate, by a simple example, the point of view which it adopts in describing the law, and the relation of this point of view to the Newtonian.

Suppose that Fig. 8 represents a crater with a house H in the middle, and that a traveller sets out to go from A to E by the shortest path. He will not necessarily pursue the path $ABHDE$ leading down to the bottom of the crater and through the house, because that may be too long. Neither will he necessarily walk right around the outside of the crater by the path $ABCDE$ because that may also be too long. By taking some such course as $ABFDE$, crossing the crater part of the way down, it is possible that he will find a path which is shorter than any other, and this is the path he will take. Suppose now that, while this is true, we know nothing about it, and that we find ourselves seated high up in an airplane watching the spectacle. Of course, I shall not see the crater, as such. Everything will appear flat. I shall see the traveller going from A to E and shall wonder why he does not go straight across, and through the house. If I have been taught in my youth that a body moves in a straight line unless a force acts upon it, I shall conclude that the house repels him. Having come to this interesting conclusion, I shall ponder over the reason why the house repels him. Possibly I shall receive a sudden inspiration which will lead me to believe that the house contains a man who is provided with a hose, which he plays on the traveller compelling him to keep away from his property. I may be able to describe the traveller's path very accurately in terms of this hose. Now, if you are philosophically inclined, you may doubt the reality of this explanation, and ask me whether I am certain that there is really a hose in that house. Of course, if you press me hard enough, I shall find difficulty in proving that there is. If you worry me enough, however, I shall get very angry with you, and call you an impracticable philosopher; but, feeling the need of saying something to your arguments, I may finally talk to you in this way: "I care not whether there is or is not a hose in that house. So long as by picturing one there, and expressing the traveller's motion in terms of its influence I can predict results which are true, I am on

perfectly firm ground. I am responsible to no man for how I think, so long as my conclusions are correct." Having said this, I shall feel that I have justified my attitude and confounded you completely. I shall begin to feel a bit of a philosopher myself; but, I shall continue to enjoy, secretly, the picture of this hose and the various details of its actions. I shall think all sorts of things about it which I shall never dare tell you lest you should laugh at me. I shall wonder what the density of the liquid composing the hose-stream may be, what its boiling point may be and so forth. Now suppose that while I am doing all this, you should bring me some observations which show that the motion of the observer was not exactly what we thought it was. The difference might be very slight, but it might be of such a nature as to upset completely the simplicity of the action which I had imagined as going on by virtue of the hose. Of course, I shall first cling to my hose, but shall modify it slightly. I shall say, "Naturally this is no ordinary kind of hose. Possibly it does not push entirely in the direction in which the liquid travels." But, I shall have to face the situation that, while the discrepancies may be small in amount, they may be large in principle, and involve such radical alterations in my notions of the mechanism of the process that the hose which I shall have to picture will be radically different from any hose which I had ever seen. I shall go on in this way, modifying and adjusting the hose, making it more and more difficult to understand; and, forgetting that the original justification for its introduction was its apparent power to explain what was observed in terms of something which I thought I knew all about, I shall soon be in the position of expending 99 per cent. of my ingenuity in trying to understand the hose, leaving only 1 per cent. for the law of the traveller. Now suppose that, while I am doing this, and am feeling rather disheartened with my success, you should come to me and say, "I have made a discovery. I do not know why the man moves as he does, neither, I think, do you, but I have found out exactly how he does move. He is moving from A to E by a path which is the shortest distance between those two points, not as the crow flies, but across a crater whose form I can describe to you." Suppose you should say this, and should then add: "Now I am going to take this statement of the law as my starting point. If there is going to be any hose in the matter, it is the hose which is going to be explained in terms of this fundamental law, and not

the fundamental law which is going to be explained in terms of the hose." I think that I should have to admit that your attitude was, at least, reasonable.

Now it is an attitude something like this which is involved in the statement of Einstein's law of gravitation. It is true that here the statement is not made entirely in empirical terms. Its structure is adorned with many beauties into which I must not enter. But as regards the law itself, the language of its statement is not unlike that for the law of our traveller. It says that a planet, in going from one point to another, moves along a path which is the longest path between those points; but, alas, for the comfort of the non-mathematical mind, that path is not one in three dimensions, but it is a path in a certain non-euclidian four dimensional space of a certain type, whose fourth dimension is time, and whose properties are expressed in such a way as to conform to certain philosophical requirements of the theory of relativity.

The philosophers of by-gone ages said that the planets moved in circles because the circle is the perfection of symmetry, and we are inclined to laugh at their attitude; but, we see that the Einstein statement of the law of gravitation, avoiding as it does the mention of forces, and stating the law only in terms of the property of the path which the planet describes, has an element of the same attitude in it. Indeed, the law of the ancient philosophers would not have been so bad if it had been a little more explicit as to the sizes of the circles, and if they had omitted the "because."

Measurement of Mercury Vapor Pressure by Means of the Knudsen Pressure Gauge. C. F. HILL. (*Phys. Rev.*, Sept., 1922.)—Very discordant results have been obtained by different experimenters for the vapor pressure of mercury from 0° to 35° C. For the temperature zero the following values of the pressure in mm. of mercury have been published: .02, .015, .00019, .00047, .0004, .000184.

"The principle upon which the Knudsen gauge depends is that the molecules of a residual gas in a partial vacuum are thrown off from a heated platinum foil, and striking a light and suitably suspended vane exert a couple, thus producing a couple which may be read by means of an optical system." The Knudsen gauge was calibrated by means of a McLeod gauge. The mercury used was distilled several times. An accuracy of 3 per cent. in the results is claimed. The pressures for 0, 10, 20, 30 and 40 degrees are .000350, .000775, .00182, .00407 and .008, respectively. G. F. S.

ON THE COMPRESSIBILITY OF MINERALS AND ROCKS AT HIGH PRESSURES.*

BY

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THE elasticity of the earth's crust is an important branch of geophysics and is a subject of interest alike to physicists and to geologists. Very little information has been available concerning the elastic properties of rocks and their constituent minerals at high pressures such as exist at considerable depths below the surface of the earth. This information is of use in connection with problems of geophysics, among which may be mentioned the tidal deformation of the earth, the elastic yielding of the crust due to loads such as mountain masses or ice-sheets, the propagation of earthquake waves, and the effect of pressure on the stability of minerals. The deformation caused by a stress acting on a sufficiently homogeneous material may be referred to two constants, in the choice of which there is considerable latitude. Thus any two of the following properties: Compressibility, Young's modulus, rigidity, or Poisson's ratio, may be chosen to represent the elastic behavior of the material. The compressibility, that is the change of volume due to pressure, is the simplest to measure, at least at high pressures, and offers the easiest avenue of approach to the study of the elasticity of the earth's crust.

In this paper are presented the results of measurements of the decrease in volume of a number of rocks and minerals at pressures up to 12,000 megabars, corresponding to a depth of about 40 kilometres below the surface of the earth. From this decrease in volume the compressibility is calculated directly, and from the compressibility there is computed, for the rocks, an approximate value of the rigidity.

METHOD.

The compressibilities were determined according to the method previously described,[1] and used for the measurement of the compressibilities of a number of solids including quartz, silica

* Communicated by Dr. Arthur L. Day, Director of Laboratory and Associate Editor of this JOURNAL.

[1] Adams, Williamson, and Johnston, "The Determination of the Compressibility of Solids at High Pressures," *J. Am. Chem. Soc.*, 1918, **41**, 12–42.

glass, halite, and calcite. Briefly, the procedure is as follows: The solid is subjected to hydrostatic pressure in a thick-walled steel cylinder, or "bomb," and is entirely surrounded by kerosene (Fig.

FIG. 1.

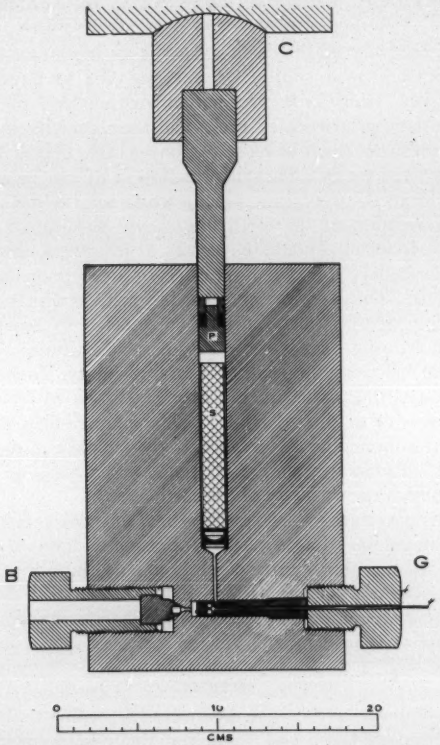
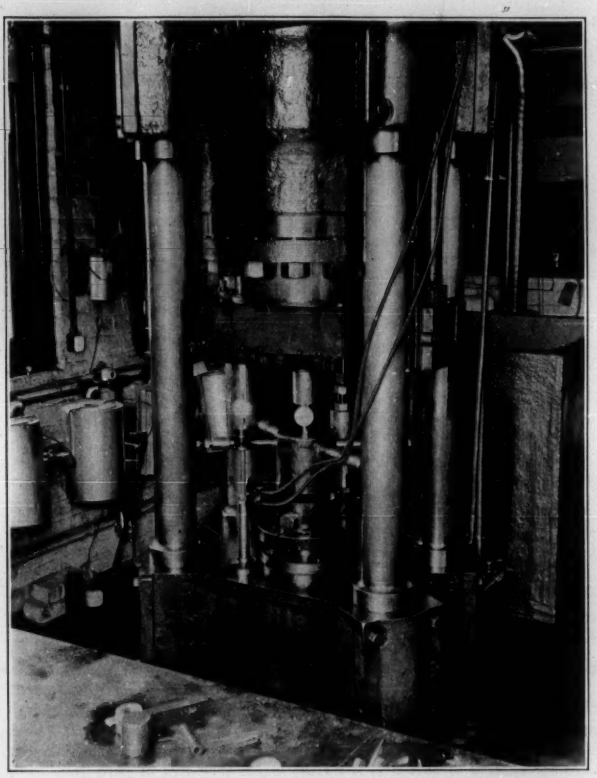


Diagram of vanadium steel bomb. The specimen, *S*, is completely surrounded by kerosene, and is subjected to pressure produced when the piston, *P*, is forced down by the motion of the press-platen, *C*. Pressure is measured by the resistance-gauge, *G*, and the volume-change of the kerosene plus specimen is determined by measuring the movement of the piston.

1) which transmits the pressure and insures that it shall be uniform in all directions, *i.e.*, purely hydrostatic. A piston provided with a leak-proof packing is forced into the bomb by means of a

hydraulic press. Its motion produces the pressure and also is a measure of the change of volume of the interior of the bomb. This volume-change is composed of three parts: (1) The decrease

FIG. 2.



Photograph of hydraulic press with bomb and appurtenances in position.

in volume of the sample itself, (2) the decrease in volume of the kerosene, and (3) the volume-change due to the distortion of the bomb and packing. The last two effects are compensated for by

an auxiliary experiment with a solid of known compressibility; the residue is the change in volume of the specimen. In effect, what is measured is the difference in compressibility of the material under investigation and that of the reference solid—soft steel—

FIG. 3.

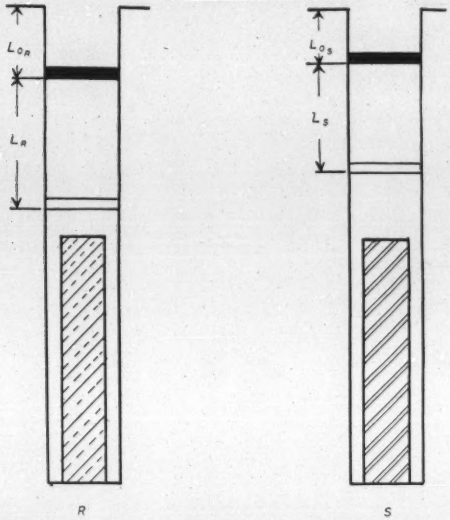


Diagram illustrating the initial and final positions of the piston in the two companion experiments: With the rock or mineral (*R*), and with the comparison steel (*S*).

the compressibility of which has been determined by a direct method,[2] and found to be 0.60×10^{-6} per megabar.[3]

[2] *Vide* P. W. Bridgman, *Proc. Am. Acad. Arts Sci.*, 1911, **47**, 366; and E. Grüneisen, *Ann. Phys.*, [4] 1910, **33**, 1262.

A recent very accurate measurement by Bridgman (*Proc. Am. Acad. Arts. Sci.*, 1923, **58**, 174) gives 0.577×10^{-6} per megabar for the compressibility of iron containing only 0.03 per cent. impurity. If this value be accepted, the values given in this paper should be lowered by 0.02×10^{-6}, but it is not impossible that the compressibility of soft steel is a trifle higher than that of pure iron.

[3] For high pressures there are three units in common use: The megabar, the atmosphere, and the kilogram per square centimetre. For several reasons, among which is the circumstance that the megabar is independent of the force of gravity, we have used that unit. One megabar = 1.0197 kg./cm.^{2} = 0.9869 atm. at Washington, D. C.

The volume-change, that is, the movement of the piston, is measured to within a few thousandths of a millimetre by a dial-gage attached to the piston; the pressure is measured to 1 megabar by an electrical resistance pressure-gage. The compressibility, β, is here defined as $-\frac{1}{V_0}\frac{dV}{dP}$ where $-dV$ is the diminution in volume for a given increment of pressure dP, and V_0 is the volume at some standard pressure and temperature. The unit of pressure being the megabar, compressibility is expressed as the relative volume-change per megabar. Thus, a compressibility of 1×10^{-6} means that the volume of the substance in question is reduced by one-millionth of its value (at the standard pressure) for each megabar increase of pressure.

Whenever possible, the materials to be investigated were fashioned into cylinders 10 cm. long and 1.6 cm. in diameter. In other cases the material in the form of coarse fragments was held in a capsule made of thin sheet copper closed at the bottom and open at the top. A diamond core-drill proved a great convenience in making cylinders of the rocks and some of the minerals. In all cases the cylinders were brought to their final size by grinding.

From the readings of pressure and piston-displacement the decrease in volume was calculated by the formula:

$$-\frac{\triangle V_r}{V_r} = -\frac{\triangle V_s}{V_s}\frac{V_s}{V_r} + \frac{A}{V_r}\left\{\triangle L(1+\alpha P) + \alpha \triangle L_0(P - P_0) - \frac{\triangle V_k}{V_k}[\triangle L_0(1+\alpha P) + Q]\right\} \quad (1)$$

which gives the fractional decrease in volume, $-\Delta V_r/V_r$ of the material at the pressure P as compared with V_r the volume at the arbitrary initial pressure P_0 (2000 megabars). In this equation $-\Delta V_s/V_s$ is the fractional decrease in volume of the reference substance (steel) as determined from $\beta_s = -\frac{\triangle V_s}{V_s}\frac{1}{P - P_0} = 0.60 \times 10^{-6}$ per megabar. The cross-section of the hole in the bomb is A at $P = 0$; $\alpha = \frac{1}{A_0}\frac{dA}{dP} = 1.2 \times 10^{-6}$ per megabar; ΔL is written for $L_r - L_s$ and is the difference of levels at the pressure P (Fig. 2); ΔL_0 is the difference of levels at the initial pressure P_0; $-\frac{\triangle V_k}{V_k}$ is the fractional decrease of volume of the kerosene between the pressures P_0 and P; and $Q = \frac{V_r - V_s}{A}$. V_r, V_s and V_k are respectively the volumes of the rock, the steel, and the kerosene at the initial pressure P_0.

It should be borne in mind that here $\Delta V_r/V_r$ and also β refer to the fractional volume-change in terms of the volume at P_0, that is, at 2000 megabars. If it is desired to refer the fractional volume-change and β to the volume at $P = 0$, the values here given for $\Delta V_r/V_r$ and for β must be multiplied by V_r/V_0, that is, by V_{2000}/V_0, the ratio of the volumes at the two pressures. It may be noted, however, that for most solids the difference between these two values of β is unimportant.

Equation (1) is in convenient form for calculating the volume-changes of the cylindrical samples, but a slight modification of (1) is more convenient in the case of the materials in granular form. Writing

$$V_r = V_{Cu} + V'_r$$

V_{Cu} being the volume of the capsule at P_0, and V'_r that of the sample at the same pressure, equation (1) becomes:

$$-\frac{\Delta V'_r}{V_r} = \frac{-(-\Delta V_{Cu})-\Delta V_s+A\left\{\Delta L(1+\alpha P)+\alpha\Delta L_0(P-P_0)-\frac{\Delta V_s}{V_s}[\Delta L_0(1+\alpha P)+Q]\right\}}{V'_r} \qquad (2)$$

which is in the form actually used for calculating the changes in volume of the granular materials.

DESCRIPTION OF MATERIALS USED.

We have studied the following minerals and rocks, of which some of the characteristic properties are given. The densities—as well as the volumes, which are necessary for the calculation of the compressibilities—were determined from the loss in weight (corrected to vacuum) when immersed in water, and are expressed as true densities. For the porous materials an approximate value of the porosity was obtained by noting the increase in weight of the material when saturated with kerosene at a pressure of several thousand megabars. The letter C after the name indicates that a cylinder was made of the material; G, that the substance was used in granular form.

Microcline. (C)—A large creamy white crystal from Topsham, Maine, kindly furnished by Professor G. P. Merrill of the U. S. National Museum. Density at 25°, 2.557. Contains 8 to 10 per cent. albite [4]; perthitic intergrowth parallel to {100}.

[4] Kindly determined by Dr. N. L. Bowen.

Oligoclase. (C)—A large white crystal from Lindstöl, Söndeled, Norway; collected by Dr. Olaf Andersen. Density at 25°, 2.638. Refractive indices and extinction angles in oriented sections correspond[4] to $Ab_{78}An_{22}$.

Labradorite. (C)—A fine specimen of the well-known material from Nain, Labrador. Obtained from Ward's Natural Science Establishment. Density at 26°, 2.695. Refractive indices and extinction angles in oriented sections correspond[4] to $Ab_{48}An_{52}$. Contains a small amount of inclusions of augite and ilmenite, which accounts for the fact that the density is about 0.015 gram/cm.^{3} higher than would be expected from the composition of the mineral.

Phlogopite Mica. (C)—Large crystal with the usual rough prismatic faces; obtained through Dr. N. L. Bowen, from Frontenac County, Ontario, Canada. A cylinder was cut with its axis parallel to the cleavage, and was held together with fine wire. Density at 25°, 2.877.

Enstatite. (CG)—Light smoky gray crystal from Espedalen, Vegaardsheien, west of Risör, Norway; collected by Dr. Olaf Andersen. Density at 25°, 3.254. Composition:[5] SiO_2, 57.28; Al_2O_3, 0.90; Fe_2O_3, 0.42; FeO, 6.43; MgO, 34.94; CaO, 0.13; Na_2O, 0.22; K_2O, 0.01; H_2O+, 0.13; TiO_2, 0.05. Minor constituents being neglected, this corresponds to: $MgSiO_3$, 88; $FeSiO_3$, 12.

Hypersthene. (G)—A large crystal from Nain, Labrador; kindly furnished by Mr. E. V. Shannon of the U. S. National Museum. Density at 29°, 3.415. Composition:[5] SiO_2, 51.81; Al_2O_3, 2.16; Fe_2O_3, 4.52; FeO, 13.96; MgO, 24.57; CaO, 1.95; Na_2O, 0.39; K_2O, 0.03; H_2O+, 0.19; TiO_2, 0.76; MnO, 0.16. Except for minor constituents, this corresponds to: $MgSiO_3$, 70; $FeSiO_3$, 30.

Diopside. (G)—Nearly colorless, transparent crystals from Ham Island, Alaska. Density at 28°, 3.257. This is the material investigated by Allen and White.[6] Composition: SiO_2, 54.65;

[5] H. S. Washington and H. E. Merwin, *Am. Mineral.*, 8, 1923. We wish to express our indebtedness to Doctor Washington for his kindness in making the analysis of this material and of several other rocks and minerals which we have used.

[6] E. T. Allen and W. P. White, *Am. J. Sci.*, 1909, **27**, 13.

Fe_2O_3, 0.13; CaO, 25.27; MgO, 18.78; Na_2O, 0.03; K_2O, 0.07; H_2O, 1.45.

Augite. (G)—Rough black crystals of the habit usual to loose, ejected volcanic augites. Obtained through Dr. H. S. Washington from Professor F. Millosevich of Rome, who collected the material at Fosso Tavolato, Alban Hills, Italy. Density at 28°, 3.373. Composition:[7] SiO_2, 48.11; Al_2O_3, 5.45; Fe_2O_3, 4.42; FeO, 3.74; MgO, 12.03; CaO, 24.50; Na_2O, 0.46; K_2O, none; H_2O+, 0.09; TiO_2, 1.19; Cr_2O_3, 0.06; MnO, 0.09.

Actinolite. (G)—A large crystal from Kragerö, Norway; collected by Dr. Olaf Andersen. Density at 28°, 3.079. Composition:[5] SiO_2, 51.86; Al_2O_3, 3.81; Fe_2O_3, 2.19; FeO, 5.97; MgO, 19.40; CaO, 10.73; Na_2O, 2.16; K_2O, 0.28; H_2O+, 0.98; TiO_2, 1.92; MnO, 0.04; (F, 0.46).

Pyrite. (C)—Large cubic crystal from Leadville, Colorado. Density at 25°, 4.992.

Diamond. (G)—Nearly colorless, rough stones[8] weighing about 1/6 carat or 0.05 g. each. Density at 25°, 3.516.

Westerly Granite. (C)—Fine-grained pink granite from Westerly, Rhode Island. Cylinder used by us was cut down from one used by Adams and Coker.[9] Density at 19°, 2.616. Porosity, 0.7 per cent. Approximate composition:[10] SiO_2, 72.26; Al_2O_3, 13.58; Fe_2O_3, 2.97; FeO, 0.75; MgO, 0.03; CaO, 1.24; Na_2O, 2.18; K_2O, 5.69; H_2O+, 0.57; H_2O-, 0.09.

Norm: Quartz, 35.0; orthoclase, 33.9; albite, 18.3; anorthite, 6.1; corundum, 1.5; hypersthene, 0.1; magnetite, 2.3; hematite, 1.4.

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[7] H. S. Washington and H. E. Merwin, *Am. Mineral*, 8, 1923.

[8] We are greatly indebted to Mr. Hans Brassler, of New York City, for lending us nearly 60 grams of rough diamonds, and we desire to record our sincere appreciation of his courtesy.

[9] "An Investigation into the Elastic Constants of Rocks, More Especially with Reference to Cubic Compressibility," Frank D. Adams and Ernest G. Coker, Carnegie Institution of Washington, Publ. No. 46.

We were very fortunate in being able to work with the identical samples used by Adams and Coker, not only the Sudbury diabase but also the Westerly granite and the New Glasgow gabbro. We are very grateful to Professor Adams for his kindness in furnishing us these materials.

[10] Analysis by I. A. Williams of rock from same locality. Quoted in Professional Paper No. 99, "Chemical Analyses of Igneous Rocks," by Henry S. Washington. Analysis No. 1, p. 165.

Mode: Quartz, 26; alkaline feldspars, mostly microcline, 70; mica, largely biotite, 4.

Stone Mountain Granite. (C)—Fine-grained gray granite from Stone Mountain near Atlanta, Georgia. Specimen taken from the block obtained by Day, Sosman, and Hostetter,[11] for the measurement of densities at high temperatures. Density at 25°, 2.633. Porosity, 0.6 per cent. The approximate composition[12] is as follows: SiO_2, 71.66; Al_2O_3, 16.05; Fe_2O_3, 0.86; FeO, not determined; MgO, 0.17; CaO, 1.07; Na_2O, 4.66; K_2O, 4.92; H_2O+, 1.00.

Norm: Quartz, 22.8; orthoclase, 28.9; albite, 39.3; anorthite, 5.3; corundum, 1.1; hypersthene, 1.7.

Mode: Quartz, 20; microcline, 40; plagioclase, $Ab_{85}An_{15}$, 30; mica, nearly all muscovite, 10.

Washington Granite. (C)—Coarse-grained gray biotite granite, somewhat gneissoid, collected by us at quarry on Tilden Street, near Connecticut Avenue, Washington, D. C. Density at 25°, 2.739. Porosity, 0.6 per cent. Composition:[13] SiO_2, 67.46; Al_2O_3, 14.22, Fe_2O_3, 1.33; FeO, 3.24; MgO, 2.85; CaO, 3.72; Na_2O, 2.88; K_2O, 2.84; H_2O+, 0.57; H_2O-, 0.12; TiO_2, 0.47; P_2O_5, 0.31; MnO, 0.08.

Norm: Quartz, 26.5; orthoclase, 16.7; albite, 24.6; anorthite, 16.4; corundum, 0.3; hypersthene, 11.3; magnetite, 1.9; ilmenite, 0.9; apatite, 0.7.

Mode (Rather difficult to determine definitely from one thin section, on account of coarseness of grain): Quartz, 32; orthoclase, 18; plagioclase, $Ab_{80}An_{20}$, 35; mica, nearly all biotite, 9; epidote, 6.

Obsidian. (C)—The well-known black material from Obsidian Cliff, Yellowstone National Park, obtained through Dr. F. E. Wright. Density at 25°, 2.333. Approximate composi-

[11] Arthur L. Day, R. B. Sosman, and J. C. Hostetter, *Am. J. Sci.*, 1914, **37**, 1–39; *Neues Jahrb. Beil. Bd.*, 1915, **40**, 119–162.

[12] Analysis of rock from same locality by R. L. Packard. *Vide* Washington, "Chemical Analyses of Igneous Rocks," U. S. Geol. Surv., Professional Paper No. 99. Analysis No. 51, p. 173.

[13] H. S. Washington, "The Granites of Washington, D. C.," *J. Wash. Acad. Sci.*, 1921, **11**, 459–470.

tion:[14] SiO_2, 74.70; Al_2O_3, 13.72; Fe_2O_3, 1.01; FeO, 0.62; MgO, 0.14; CaO, 0.78; Na_2O, 3.90; K_2O, 4.02; H_2O+, 0.62; FeS_2, 0.40.

Norm: Quartz, 34.9; orthoclase, 23.4; albite, 33.0; anorthite, 3.9; corundum, 1.5; hypersthene, 0.7; magnetite, 1.4.

Mode: All glass.

Gabbro. (C)—This specimen, from New Glasgow, Quebec, is identical with the material used by Adams and Coker and described in the publication cited. It is a dark greenish color, coarse-grained, and somewhat variably mottled. Density at 23°, 3.106. Composition:[15] SiO_2, 44.67; Al_2O_3, 20.74; Fe_2O_3, 2.13; FeO, 4.10; MgO, 16.97; CaO, 9.90; Na_2O, 1.15; K_2O, 0.59; H_2O+, 0.15; H_2O-, 0.06; TiO_2, 0.09; P_2O_5, none; MnO, n.d.

Norm: Orthoclase, 3.3; albite, 10.0; anorthite, 49.2; hypersthene, 1.4; olivine, 33.1; magnetite, 3.0; ilmenite, 0.2.

Mode: There appears to be 25 per cent. plagioclase, $Ab_{35}An_{65}$; 40 per cent. hypersthene, and 35 per cent. olivine (Adams and Coker[16] report: Not over one-quarter plagioclase, the remaining minerals being augite and rhombic pyroxene in equal amounts.) Neither of these estimates agrees with the norm, which is probably due to lack of uniformity of the rock.

Sudbury Diabase. (C)—This was part of one of the cylinders used by F. D. Adams and E. G. Coker in their investigation of the elastic constants of certain rocks.[17] This rock is a moderately coarse-grained diabase. It comes from the Murray mine near Sudbury, Ontario, and is fully described by Adams and Coker. Density at 28°, 3.002. Porosity, 0.05 per cent. Composition:[18] SiO_2, 48.40; Al_2O_3, 16.07; Fe_2O_3, 2.10; FeO, 11.91; MgO, 5.68; CaO, 7.87; Na_2O, 4.61; K_2O, 0.48; H_2O+, 0.38; H_2O-, 0.09; TiO_2, 2.53; P_2O_5, 0.16; MnO, 0.11.

Norm: Orthoclase, 2.8; albite, 27.5; anorthite, 22.0; nephelite, 6.1; diopside, 13.6; olivine, 17.4; magnetite, 3.0; ilmenite, 4.9; apatite, 0.3.

Mode: Plagioclase, $Ab_{35}An_{65}$, 70; olivine, 15; augite, 10; magnetite, 5.

[14] Analysis by J. E. Whitfield of an obsidian from same locality. *Vide* H. S. Washington, "Chemical Analyses of Igneous Rocks," U. S. Geol. Surv., Professional Paper No. 99. Analysis No. 18, p. 63.

[15] H. S. Washington, unpublished.

[16] Adams and Coker, *op. cit.*, p. 55.

[17] *Op. cit.*

[18] H. S. Washington, unpublished.

Palisade Diabase. (C)—Fine-grained diabase from quarry one mile north of Granton, New Jersey. This is a specimen of the rock used by Day, Sosman, and Hostetter in their investigation of mineral and rock densities at high temperatures.[19] Density at 25°, 2.975. Porosity, 0.06 per cent. This rock has been described by Lewis[20] and probably corresponds closely to his analyses XII and XIII, p. 121 of his report. Analysis XIII is as follows:[21] SiO_2, 50.34; Al_2O_3, 15.23; Fe_2O_3, 2.82; FeO, 11.17; MgO, 5.81; CaO, 9.61; Na_2O, 2.93; K_2O, 1.02; H_2O+, 0.07; H_2O-, 0.19; TiO_2, 1.56; P_2O_5, 0.20; MnO, 0.14.

Norm: Orthoclase, 6.1; albite, 24.6; anorthite, 25.3; diopside, 18.0; hypersthene, 13.6; olivine, 5.8; magnetite, 4.2; ilmenite, 3.0; apatite, 0.3.

Mode: Plagioclase, $Ab_{40}An_{60}$, 49; augite, 50; magnetite, 1.

Basalt. (C)—Dense, black, non-porphyritic basalt from Hatfield and Weldon's quarry, Scotch Plains, New Jersey. Density at 26°, 2.911. Porosity, 0.9 per cent. Its composition is probably represented by analysis V of Lewis' report.[22] This analysis is as follows: SiO_2, 51.84; Al_2O_3, 15.11; Fe_2O_3, 1.78; FeO, 8.31; MgO, 7.27; CaO, 10.47; Na_2O, 1.87; K_2O, 0.34; H_2O+, 1.33; H_2O-, 0.56; TiO_2, 1.22; P_2O_5, 0.13; MnO, 0.09.

Norm: Quartz, 5.0; orthoclase, 2.2; albite, 16.2; anorthite, 31.4; diopside, 16.3; hypersthene, 22.1; magnetite, 2.6; ilmenite, 2.3; apatite, 0.3.

Mode: Plagioclase, $Ab_{45}An_{55}$, 30; augite, 50; glass, 20.

Marble. (C)—Pure white, coarse-grained material from Ule Creek, Gunnison County, Colorado. Part of a large piece given to us by Professor G. P. Merrill. Porosity, 0.8 per cent. Density at 25°, 2.708.

Serpentine. (*Talc schist*) (C)—This material is commonly called "Alberene Stone," and comes from Alberene, Virginia. Density at 19°, 2.875.

It consists of about 50 per cent. talc, 15 per cent. calcite, 15 per cent. serpentine, and smaller amounts of chlorite and magnetite.

[19] Day, Sosman, and Hostetter, *op. cit.*

[20] Geol. Surv. New Jersey, Ann. Report, 1907, 97–168.

[21] *Cf.* H. S. Washington, "Chemical Analyses of Igneous Rocks," U. S. Geol. Surv., Professional Paper No. 99. Analysis No. 15, p. 607.

[22] *Op. cit.*, p. 159. *Cf.* H. S. Washington, "Chemical Analyses of Igneous Rocks," U. S. Geol. Surv., Professional Paper No. 99. Analysis No. 22, p. 641.

Plate Glass. (C)—A cylinder was ground from a piece of plate glass. Density at 25°, 2.530. This particular glass was not analyzed, but is known to have approximately the following composition: SiO_2, 73; CaO, 12; Na_2O, 15.

Cast Iron. (C)—This specimen was soft gray cast iron. Density at 23°, 7.193.

EXPERIMENTAL RESULTS.

Minerals.—The results of the measurements on the various minerals are presented in Tables I and II, and summarized in Table III. Table I contains the results obtained with the minerals which were used in the form of cylinders. In the columns of this table are successively: The pressure in megabars; the corresponding value of ΔL, that is, of $L_r - L_s$, the difference in piston-displacements in the experiment with the comparison steel and in the experiment with the material under investigation; ΔL (corrected), that is the value of the term enclosed within the brackets in equation (1); $-(\Delta V_r/V_r - \Delta V_s/V_r)$, the difference between the relative volume-changes of the steel and the given material; and $-(\Delta V_r/V_r)$, the relative decrease in volume of the mineral at the given pressure. The figures in the sixth and seventh columns will be explained later. For the measurements in this table the volume of the comparison steel cylinder was 20,054 cubic millimetres at $P = 1$ megabar. Below each sub-heading are given: The value of ΔL_0 for that experiment; V, the volume of the mineral at $P = 1$; and Q, (that is, $(V_r - V_s)/A$), where V_r and V_s are the volumes at $P = 2000$. Table II, which is for the minerals used in the granular form, is similar to Table I except that the figures corresponding to column 4 of Table I do not appear. This is due to the fact that equation (2) is the one used for calculating the volume-changes of the minerals in granular form. The volume of the comparison steel was 10,674 mm.^{3} at $P = 1$, except for the series with diamond, in which case the volume was 9879 mm.^{3} ΔL_0 and Q have the same meaning as in Table I; V' is the volume of mineral itself; V_{Cu} is the volume of the copper capsule, and V is the sum of the two, all at $P = 1$.

The relative volume-change at the given pressure as obtained[23] from equation (1) or (2) is shown in column 5 of Table I and

[23] For further details, including the compressibility of the kerosene used, see Adams, Williamson, and Johnston, *op. cit.*

TABLE I.

Experimental Results for Minerals in Cylindrical Form. Volume of Reference Steel, 20,054 mm.³ at P = 0.

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_r)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			OLIGOCLASE.			
$\Delta L_0 = -0.12$	$V = 20{,}122$	$Q = 0.09$	$10^4 a = -.2$	$10^6 b = 1.725$	$10^{10} c = 0.057$	
12,000	0.931	0.938	0.01080	0.01630	0.01666	1.4
11,000	.830	.836	964	1504	1504	.0
10,000	.735	.739	851	1331	1341	−1.0
9,000	.658	.660	760	1180	1178	.2
8,000	.555	.556	641	1001	1011	−1.0
7,000	.475	.475	547	847	846	.1
6,000	.372	.372	428	666	679	−1.3
5,000	.282	.282	325	505	510	− .5
4,000	.196	.195	224	344	341	.3
3,000	.102	.101	116	176	170	.4
2,000	0.000	0.000	0.00000	0.00000	−0.00002	.2
			OLIGOCLASE.			
$\Delta L_0 = 0.11$	$V = 20{,}122$	$Q = 0.09$	$10^4 a = .7$	$10^6 b = 1.726$	$10^{10} c = 0.080$	
7,840	0.523	0.551	0.00635	0.00989	0.00994	− .5
6,860	.448	.473	545	839	834	.5
5,880	.356	.377	434	669	671	− .2
4,900	.276	.292	336	512	508	.4
3,920	.186	.197	227	345	342	.3
2,940	.100	.105	121	180	176	.4
1,960	0.000	0.000	0.00000	0.00000	0.00007	− .7
			OLIGOCLASE.			
$\Delta L_0 = 0.70$	$V = 20{,}107$	$Q = 0.02$	$10^4 a = 0.0$	$10^6 b = 1.691$	$10^{10} c = 0.115$	
12,000	0.700	0.826	0.00971	0.01571	0.01576	− .5
10,000	.578	.688	809	1289	1279	1.0
8,000	.427	.516	607	967	973	− .6
6,000	.287	.353	415	655	658	− .3
4,000	.144	.183	215	335	334	.1
3,000	.071	.091	107	167	168	− .1
2,000	0.000	0.000	0.00000	0.00000	0.00000	.0
			LABRADORITE.			
$\Delta L_0 = -0.08$	$V = 20{,}216$	$Q = 0.54$	$10^4 a = .5$	$10^6 b = 1.531$	$10^{10} c = 0.104$	
12,000	0.645	0.724	0.00837	0.01437	0.01432	.5
11,000	.576	.649	750	1290	1298	− .8
10,000	.518	.587	678	1158	1161	− .3
9,000	.474	.535	619	1039	1026	1.3
8,000	.398	.453	525	885	886	.1
7,000	.342	.389	451	751	745	.6
6,000	.262	.301	348	588	601	−1.3
5,000	.202	.234	271	451	455	− .4
4,000	.143	.167	193	313	307	.6
3,000	.079	.091	105	165	157	.8
2,000	0.000	0.000	0.00000	0.00000	0.00005	− .5

TABLE I.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_T/V_T - \Delta V_S/V_T)$	$-(\Delta V_T/V_T)$ (obs.)	$-(\Delta V_T/V_T)$ (calc.)	Diff. $\times 10^4$
			LABRADORITE.			
$\Delta L_0 = 0.015$ $V = 20{,}216$ $Q = 0.54$ $10^4 a = .4$ $10^6 b = 1.512$ $10^{10} c = 0.128$						
7,840	0.354	0.419	0.00485	0.00839	0.00849	−1.0
6,860	.309	.366	423	717	715	.2
5,880	.252	.301	348	583	578	.5
4,900	.187	.226	262	438	438	.0
3,920	.135	.164	189	307	296	1.1
2,940	.056	.071	082	141	151	−1.0
1,960	0.000	0.000	0.00000	0.00000	0.00004	− .4
			LABRADORITE.			
$\Delta L_0 = -0.06$ $V = 20{,}216$ $Q = 0.54$ $10^4 a = -0.2$ $10^6 b = 1.482$ $10^{10} c = 0.110$						
12,000	0.569	0.647	0.00756	0.01356	0.01371	−1.5
10,000	.494	.562	660	1140	1114	2.6
8,000	.359	.414	484	844	848	− .4
6,000	.228	.270	316	556	573	−1.7
4,000	.120	.145	170	290	290	.0
3,000	.061	.074	87	147	145	.2
2,000	0.000	0.000	0.00000	0.00000	−0.00002	.2
			MICROCLINE.			
$\Delta L_0 = 0.07$ $V = 20{,}074$ $Q = -0.16$ $10^4 a = .3$ $10^6 b = 1.880$ $10^{10} c = 0.128$						
12,000	0.998	1.000	0.01154	0.01754	0.01755	− .1
11,000	.915	.915	1056	1596	1591	.5
10,000	.817	.816	942	1422	1425	− .3
9,000	.728	.726	838	1258	1264	− .6
8,000	.631	.628	725	1085	1085	.0
7,000	.532	.527	609	909	911	− .2
6,000	.428	.423	488	728	734	− .6
5,000	.334	.330	382	562	555	.7
4,000	.225	.222	256	376	374	.2
3,000	.117	.115	133	193	190	.3
2,000	0.000	0.000	0.00000	0.00000	0.00003	− .3
			MICROCLINE.			
$\Delta L_0 = -0.20$ $V = 20{,}074$ $Q = 0.13$ $10^4 a = 1.0$ $10^6 b = 1.865$ $10^{10} c = 0.095$						
7,840	0.653	0.622	0.00718	0.01072	0.01075	− .3
6,860	.554	.526	607	901	902	− .1
5,880	.449	.423	489	724	727	− .3
4,900	.348	.327	378	554	551	.3
3,920	.243	.228	263	381	372	.9
2,940	.128	.119	137	196	192	.4
1,960	0.000	0.000	0.00000	0.00000	0.00010	−1.0

TABLE I.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_r)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			ENSTATITE.			
$\Delta L_0 = -0.47$ $V = 20{,}065$ $Q = -0.05$ $10^4 a = 1.4$ $10^6 b = 1.000$						
12,000	0.434	0.356	0.00411	0.01011	0.01014	− .3
11,000	.390	.316	367	907	914	− .7
10,000	.355	.285	329	809	814	− .5
9,000	.322	.257	297	717	714	.3
8,000	.281	.222	256	616	614	.2
7,000	.249	.197	223	523	514	.9
6,000	.202	.157	181	421	414	.7
5,000	.160	.124	143	323	314	.9
4,000	.109	.082	095	215	214	− .1
3,000	.060	.046	053	113	114	.1
2,000	0.000	0.000	0.00000	0.00000	0.00014	−1 4
			ENSTATITE.			
$\Delta L_0 = -0.60$ $V = 20{,}065$ $Q = -0.05$ $10^4 a = -.6$ $10^6 b = 1.080$						
7,840	0.311	0.238	0.00275	0.00629	0.00629	.0
6,860	.267	.201	232	526	523	.3
5,880	.214	.158	182	417	418	− .1
4,900	.158	.113	130	306	312	− .6
3,920	.118	.086	099	217	206	1.1
2,940	.043	.024	028	087	100	−1.3
1,960	0.000	0.000	0.00000	0.00000	−0.00006	.6
			MICA.			
$\Delta L_0 = 0.54$ $V = 18{,}468$ $Q = -7.14$ $10^4 a = 1.4$ $10^6 b = 2.270$ $10^{10} c = 0.174$						
7,840	1.450	0.735	0.00923	0.01277	0.01289	−1.2
6,860	1.264	.631	796	1090	1085	.5
5,880	1.059	.516	648	883	878	.5
4,900	.835	.393	493	669	667	.2
3,920	.603	.277	348	456	453	.3
2,940	.323	.147	184	243	235	.8
1,960	0.000	0.000	0.00000	0.00000	0.00014	−1.4
			PYRITE.			
$\Delta L_0 = -0.91$ $V = 20{,}163$ $Q = 0.33$ $10^4 a = 0.8$ $10^6 b = 0.710$						
7,840	0.131	0.061	0.00070	0.00424	0.00426	− .2
6,860	.109	.049	56	350	356	− .6
5,880	.100	.048	55	290	286	.4
4,900	.081	.039	45	221	217	.4
3,920	.057	.026	30	148	147	− .1
2,940	.037	.020	23	82	78	.4
1,960	0.000	0.000	0.00000	0.00000	0.00008	− .8

TABLE II.

Experimental Results for Minerals in Granular Form. Volume of Reference Steel 10,674 (9879 for Diamond).

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			ACTINOLITE.		
$\Delta L_0 = 0.18$ $V = 10{,}802$ $V' = 9913$ $V_{Cu} = 889$ $Q = 0.52$ $10^4 a = 1.7$ $10^6 b = 1.320$					
12,000	0.199	0.312	0.01317	0.01337	−2.0
10,000	.167	.266	1093	1073	2.0
8,000	.117	.191	818	809	.9
6,000	.069	.131	540	545	− .5
4,000	.031	.067	275	281	− .6
3,000	.027	.047	169	149	2.0
2,000	0.000	0.000	0.00000	0.00017	−1.7
			AUGITE.		
$\Delta L_0 = 0.00$ $V = 10{,}837$ $V' = 9965$ $V_{Cu} = 872$ $Q = 0.665$ $10^4 a = 3.4$ $10^6 b = 1.035$					
12,000	0.106	0.206	0.01064	0.01069	− .5
10,000	.080	.169	859	862	− .3
8,000	.063	.137	668	655	1.3
6,000	.029	.085	431	448	−1.7
4,000	.025	.058	252	241	1.1
3,000	.030	.048	171	138	3.3
2,000	0.000	0.000	0.00000	0.00034	−3.4
			DIOPSIDE.		
$\Delta L_0 = 0.41$ $V = 10{,}847$ $V' = 9976$ $V_{Cu} = 871$ $Q = 0.69$ $10^4 a = 5.7$ $10^6 b = 1.089$					
12,000	0.063	0.237	0.01136	0.01146	−1.0
10,000	.046	.199	928	928	.0
8,000	.033	.161	724	710	1.4
6,000	.007	.104	474	493	−1.9
4,000	.020	.077	297	275	2.2
3,000	.038	.068	217	166	5.1
2,000	0.000	0.000	0.00000	0.00057	−5.7
			HYPERSTHENE.		
$\Delta L_0 = 0.40$ $V = 10{,}675$ $V' = 9901$ $V_{Cu} = 774$ $Q = -0.03$ $10^4 a = -0.2$ $10^6 b = 1.008$					
12,000	0.123	0.183	0.01022	0.01006	1.6
10,000	.082	.135	789	804	−1.5
8,000	.058	.102	593	603	−1.0
6,000	.040	.073	407	401	.6
4,000	.015	.035	201	200	.1
3,000	.006	.017	99	99	.0
2,000	0.000	0.000	0.00000	−0.00002	.2
			ENSTATITE.		
$\Delta L_0 = 0.30$ $V = 10{,}688$ $V' = 9910$ $V_{Cu} = 778$ $Q = 0.03$ $10^4 a = -0.4$ $10^6 b = 1.043$					
12,000	0.144	0.197	0.01054	0.01039	1.5
10,000	.101	.148	819	830	−1.1
8,000	.072	.111	614	622	− .8
6,000	.042	.071	403	413	−1.0
4,000	.024	.041	215	205	1.0
3,000	.008	.017	99	100	− .1
2,000	0.000	0.000	0.00000	−0.00004	.4

column 4 of Table II. The arbitrary initial pressure, P_0, is 2000 megabars: The values in the two columns mentioned therefore indicate, in cm.^{3}, how much one cm.^{3} (at 2000 megabars) of the material decreases in volume when the pressure is raised from 2000 megabars to the given pressure. It may be observed that this volume-change is nearly but not quite proportional to the pressure-

TABLE II.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
		DIAMOND.			
$\Delta L_0=0.04$ $V=9080$ $V'=9891$ $V_{Cu}=811$ $Q=0.09$ $10^4a=-1.6$ $10^6b=0.180$					
12,000	−0.183	−0.166	0.00151	0.00164	−1.3
11,000	− .151	− .134	174	146	2.8
10,000	− .138	− .121	151	128	2.3
8,000	− .126	− .112	58	92	−3.4
6,000	− .089	− .078	30	56	−2.6
4,000	− .045	− .038	18	20	− .2
3,000	− .023	− .019	11	2	.9
2,000	0.000	0.000	0.00000	−0.00016	1.6
		DIAMOND.			
$\Delta L_0=0.20$ $V=9080$ $V'=9891$ $V_{Cu}=811$ $Q=0.09$ $10^4a=-0.4$ $10^6b=0.205$					
12,000	−0.193	−0.151	0.00189	0.00201	−1.2
11,000	− .165	− .124	200	181	1.9
10,000	− .150	− .112	174	160	1.4
8,000	− .131	− .099	92	119	−2.7
6,000	− .084	− .061	74	78	− .4
4,000	− .042	− .027	46	37	.9
3,000	− .025	− .017	14	17	− .3
2,000	0.000	0.000	0.00000	−0.00004	.4

difference. As explained in a previous publication,[24] the volume-change as a function of pressure is represented by the equation:

$$-\frac{\Delta V_r}{V_r} = a + b\,(P - P_0) - c(P - P_0)^2 \qquad (3)$$

in which a, b, and c are constants. The value of c is small in any case, and is zero when the volume-change is proportional to the pressure. For each series of measurements the coefficients, a, b, c, of equation (3) were calculated by the method of least squares; with these values of the coefficients, the values of $-(\Delta V_r/V_r)$ were calculated from the equation and listed in column 6 of Table I and column 5 of Table II. How well the equation fits the data is shown by comparing the observed and calculated volume-changes. The differences between $-(\Delta V_r/V_r)$ *obs.* and $-(\Delta V_r/V_r)$ *calc.*

[24] Adams, Williamson, and Johnston, *op. cit.*, p. 36.

are shown in the last columns of Tables I and II. These differences average but little more than 1×10^{-4} and correspond to an average error of about 0.01 mm. in the measurement of piston-displacement. The error is greater with the granular material than with the cylinders, mainly because of the fact that less material could be used in the former case. The values of a, b, and c for each series of measurements are included in Tables I and II. In those cases in which c is not given it is equal to zero.

TABLE III.

Summary of Results for Minerals.

Mineral.	Composition.	Coeffs. of Eq. (4)		Compressibility. $\beta \times 10^6$		
		$b \times 10^6$	$c \times 10^{12}$	at 0	at 2000	at 10,000
Oligoclase	$Ab_{78}.An_{22}$	1.712	0.083	1.74	1.71	1.58
Labradorite	$Ab_{48}.An_{52}$	1.508	0.109	1.55	1.51	1.34
(Anorthite)*	$CaO.Al_2O_3.2SiO_2$	.....	.....	....	(1.1$_9$)	(1.1$_0$)
(Andesine)*	$Ab_2.An_1$	.....	.....	....	(1.6$_3$)	(1.4$_9$)
(Labradorite)*	$Ab_1.An_2$	.....	.....	....	(1.3$_7$)	(1.3$_0$)
(Albite)*	$Na_2O.Al_2O_3.6SiO_2$	.....	.....	....	(1.9$_0$)	(1.6$_9$)
Microcline	$Or_{91}.Ab_9$	1.875	0.121	1.92	1.88	1.68
(Orthoclase)	$K_2O.Al_2O_3.6SiO_2$	.....	.....	(1.92)	(1.88)	(1.68)
Enstatite	$(MgSiO_3)_{88}(FeSiO_3)_{12}$	1.031	.....	1.03	1.03	1.03
Hypersthene	$(MgSiO_3)_{70}(FeSiO_3)_{30}$	1.008	.....	1.01	1.01	1.01
Diopside	$CaO.MgO.2SiO_2$	1.089	.....	1.09	1.09	1.09
Augite	.....	1.035	.....	1.04	1.04	1.04
Actinolite	.....	1.320	.....	1.32	1.32	1.32
Phlogopite Mica	.....	2.270	0.174	2.34	2.27	1.99
Pyrite	FeS_2	0.710	.....	0.71	0.71	0.71
Diamond	C	0.193	.....	0.18	0.18	0.18
[Quartz]†	SiO_2	2.625	0.194	2.70	2.63	2.31
[Silica glass]†	SiO_2	3.156	.....	3.1	3.1	....
[Calcite]†	$CaCO_3$	1.390	.....	1.39	1.39	1.39
[Halite]†	NaCl	4.006	0.295	4.12	4.01	3.53

* By interpolation.

† Data from previous paper, Adams, Williamson and Johnston, *J. Am. Chem. Soc.*, 1919, 41, 39.

The experimental results for minerals are summarized in Table III, the third and fourth columns of which contain the weighted averages of b and c, respectively, for each of the minerals. Where more than one series of measurements was obtained for the same mineral, the average b was obtained by weighting the longer runs twice as much as the shorter ones; for c the ratio was 4 to 1. The coefficient, b, is equal to the compressibility, β, at P_0 (2000 megabars), and the coefficient, c, is a measure of the change of compressibility with pressure. At any pressure the value of β may be obtained by differentiating equation (3); *viz.*,

$$\beta = b - 2c(P - P_0) \qquad (4)$$

Table III shows the values of β at 0, 2000 and 10,000 megabars for the various minerals investigated, as obtained from b and c by means of equation (4). The compressibilities of albite and anorthite were obtained by graphical extrapolation of the values for oligoclase and labradorite, the assumption being made that for the plagioclase series the compressibility is a linear function of the composition[25] by volume (which in this case does not differ by an important amount from the composition by weight), and that the compressibility of anorthite is the same at 10,000 megabars as at 2000. The compressibility of diamond, according to the present measurements, is 0.19×10^{-6} per megabar, which agrees fairly well with the value, 0.16×10^{-6}, previously obtained by L. H. Adams [26] on somewhat less favorable material. The mean of these two determinations, rounded off to two significant figures, is 0.18×10^{-6}, and is thought to be the most probable value. The results of previous measurements,[27] carried out in this laboratory, on the three minerals, quartz, calcite, and halite, and on silica glass, are included in Table III.

From an inspection of Table III it may be seen that the compressibility of the minerals investigated varies from 0.18×10^{-6} for diamond up to 4.12×10^{-6} for halite (at $P = 0$). Diamond, it may be mentioned, has the lowest compressibility of any known substance. As a rule, the higher the compressibility the more the change in compressibility with pressure. The compressibility of quartz, for example, falls off from 2.70×10^{-6} at $P = 0$ to 2.31×10^{-6} at $P = 10{,}000$, a decrease of 14 per cent. for 10,000 megabars. Enstatite, on the other hand—$\beta = 1.03 \times 10^{-6}$—shows, within the error of experiment, a constant compressibility over the whole pressure range. A dividing line can be drawn at $\beta = 1.5 \times 10^{-6}$, below which the compressibility changes so little as to escape detection by the present method, and above which the change of compressibility with pressure is easily measured. The single exception is silica glass which apparently shows constant compressibility—$\beta = 3.1 \times 10^{-6}$—over the range of pressure used, which in that case was from 2000 to 8000 megabars.

The compressibility of the microperthite is almost identical with that of the albite, and since there is no reason to suppose that

[25] *Vide* Adams, Williamson, and Johnston, *op. cit.*, p. 40.

[26] *J. Wash. Acad. Sci.*, 1921, 11, 45–50.

[27] Adams, Williamson, and Johnston, *op. cit.*, p. 39.

microcline and orthoclase would differ in compressibility, the value assigned to the impure microcline in Table III, namely, 1.92, 1.88 and 1.68 at $P = 0$, $P = 2000$, and $P = 10{,}000$, respectively, may be taken for the compressibility of pure orthoclase. Again, the compressibilities of the enstatite and of the hypersthene are practically the same—1.03 against 1.01—although the former contains (essentially) 12 per cent. $FeSiO_3$ and the latter 30 per cent. The difference in compressibility is within the error of experiment; and therefore the compressibility of any mineral of the series, that is of any ordinary enstatite, bronzite, or hypersthene, is probably not far from 1.02×10^{-6}, the average of the above values.

It is an interesting and significant fact that the four pyroxenes investigated—enstatite, hypersthene, diopside, and augite—have compressibilities lying within a very narrow range, that is from 1.01×10^{-6} to 1.09×10^{-6}. They are less compressible than the amphibole, actinolite ($\beta = 1.32 \times 10^{-6}$), and much less compressible than any of the feldspars except anorthite or anorthite-rich plagioclase. For a first approximation the compressibilities of the silicate minerals (pure SiO_2 is excepted) may be calculated from certain values which may be assigned to the constituent oxides. These values, which may be called the "compressibilities in combination," and which are not necessarily the same as the compressibilities of the actual oxides, are as follows: SiO_2, 1.4; Al_2O_3, 0.8; FeO (and Fe_2O_3), 0.5; MgO, 0.7; CaO, 0.8; K_2O and Na_2O, 6.0. The number of molecules (by weight) of each oxide is multiplied by the corresponding factor, and the sum of these products is divided by the total number of molecules.[28] The calculation agrees very closely with the observed values for all of the pyroxenes and the feldspars, but leads to too low a value for actinolite, probably on account of the water present in this mineral and on account of the inherently greater compressibility of the amphibole structure as compared with the pyroxene structure. A glance at the above list of factors for the various oxides shows the tremendous effect of soda and potash in increasing the compressibility of compounds.

By the use of the above factors the compressibility of the important mineral olivine is readily calculated to be 0.93×10^{-6}, and

[28] Probably greater accuracy could be attained by taking account of *molecular* volumes, and *molecular* compressibilities, of the oxides in combination, but there are insufficient data to justify such a calculation.

it is highly improbable that this value is in error by as much as 10 per cent.

Rocks.—The rocks were always investigated in the form of cylinders; granular material was not used, as was done in the case of some of the minerals. For the first group of experiments the cylinders were surrounded by kerosene and subjected to pressure just as the minerals were. In Table IV are given the experimental data including the volume-changes at each pressure as determined by the aid of equation (1). In this table the column headings and other symbols have the same significance as in Table I. The decrease of volume, $-(\Delta V_r/V_r)$, as before is nearly but not quite proportional to the increase of pressure, $P - P_0$, and follows equation (3) with sufficient accuracy. Rocks, unlike minerals, are usually porous; and before commenting on the compressibility of the rocks, we shall consider the effect of pressure on porous materials, and shall present the results of further experiments.

Enclosed Rocks; "Inner" and "Outer" Compressibility.—When a porous material, such as a rock, is in contact with the liquid which transmits the pressure, the liquid may penetrate the pores and act directly on the individual grains or non-porous parts of the solid. The compressibility then would be dependent mainly on the individual compressibilities of the constituents of the solid. The results of Table IV yield such a type of compressibility. According to the method of calculation used, the volume-change of the kerosene, whether outside the rock or within its pores, is eliminated, leaving for this set of data only the volume-change of the rock grains, and including no volume-decrease due to the packing together of the individual grains. This type of compressibility of a porous solid may be called its *"inner" compressibility.*

Now suppose that, on the other hand, the solid is covered by a thin, yielding, but impervious coating. The decrease in volume of the material when subjected to pressure would now include the effect of the packing together of the homogeneous parts of the solid—that is the effect of reduction in volume of the pore-space. As an example, consider the effect of pressure on a piece of sponge, (1) in contact with and surrounded by a liquid, and (2) also surrounded by the liquid, but covered with a close-fitting envelope of thin sheet rubber. The volume-change in the first case (assuming, of course, that the volume-change of the liquid is eliminated) is due entirely to the compressibility of the fibres of which the

TABLE IV.

Experimental Results for Unenclosed Rocks. Volume of Reference Steel, 20,054.

Pressure Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_r)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			MARBLE.			
$\Delta L_0 = 0.08$ $V = 20{,}078$ $Q = -0.06$ $10^4 a = 1.8$ $10^6 b = 1.412$						
12,000	0.702	0.716	0.00826	0.01426	0.01430	− .4
11,000	.620	.632	729	1269	1289	−2.0
10,000	.569	.580	670	1150	1147	.3
9,000	.510	.518	598	1018	1006	1.2
8,000	.435	.442	510	870	865	.5
7,000	.376	.381	440	740	724	1.6
6,000	.295	.297	343	583	583	.0
5,000	.226	.228	261	441	441	.0
4,000	.158	.160	185	305	300	.5
3,000	.089	.089	103	163	159	.4
2,000	0.000	0.000	0.00000	0.00000	0.00018	−1.8
			MARBLE.			
$\Delta L_0 = 0.15$ $V = 20{,}078$ $Q = -0.06$ $10^4 a = 1.0$ $10^6 b = 1.410$						
7,840	0.389	0.404	0.00466	0.00820	0.00839	−1.9
6,860	.338	.351	405	699	701	− .2
5,880	.277	.288	332	567	563	.4
4,900	.213	.221	255	431	425	.6
3,920	.146	.151	177	295	286	.9
2,940	.084	.086	099	158	148	1.0
1,960	0.000	0.000	0.00000	0.00000	0.00010	−1.0
			SUDBURY DIABASE.			
$\Delta L_0 = -0.58$ $V = 20{,}118$ $Q = 0.15$ $10^4 a = 1.0$ $10^6 b = 1.237$						
12,000	0.632	0.570	0.00657	0.01257	0.01247	1.0
11,000	.560	.500	576	1116	1123	− .7
10,000	.505	.449	517	997	999	− .2
9,000	.449	.396	456	876	876	.0
8,000	.389	.341	393	753	752	.1
7,000	.327	.285	328	628	628	.0
6,000	.264	.227	262	502	505	− .3
5,000	.203	.173	199	379	381	− .2
4,000	.150	.129	148	268	257	.9
3,000	.084	.071	082	142	133	.9
2,000	0.000	0.000	0.00000	0.00000	0.00010	−1.0
			SUDBURY DIABASE.			
$\Delta L_0 = -0.16$ $V = 20{,}118$ $Q = 0.15$ $10^4 a = -.9$ $10^6 b = 1.273$						
12,000	0.581	0.581	0.00668	0.01268	0.01264	.4
11,000	.514	.513	591	1131	1137	− .6
10,000	.461	.460	529	1009	1009	.0
9,000	.412	.410	472	892	882	1.0
8,000	.348	.346	398	758	755	.3
7,000	.288	.285	328	628	627	.1
6,000	.215	.213	245	485	500	−1.5
5,000	.164	.162	186	366	373	− .7
4,000	.112	.111	128	248	246	.2
3,000	.051	.050	058	118	118	.0
2,000	0.000	0.000	0.00000	0.00000	−0.00009	.9

TABLE IV.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_r)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
PALISADE DIABASE.						
$\Delta L_0 = -0.40$ $V = 20{,}082$ $Q = -0.05$ $10^4 a = 1.8$ $10^6 b = 1.442$						
12,000	0.812	0.752	0.00867	0.01467	0.01460	.7
11,000	.716	.657	758	1298	1316	−1.8
10,000	.645	.589	679	1159	1171	−1.2
9,000	.579	.526	606	1026	1027	− .1
8,000	.511	.462	533	893	883	1.0
7,000	.431	.388	447	747	739	1.2
6,000	.348	.310	357	597	595	.2
5,000	.275	.244	281	461	451	1.0
4,000	.183	.160	184	304	306	− .2
3,000	.101	.089	103	163	162	.1
2,000	0.000	0.000	0.00000	0.00000	0.00018	−1.8
PALISADE DIABASE.						
$\Delta L_0 = 0.93$ $V = 20{,}082$ $Q = -0.05$ $10^4 a = 1.5$ $10^6 b = 1.428$						
12,000	0.567	0.713	0.00823	0.01423	0.01443	−2.0
11,000	.514	.652	752	1292	1300	− .8
10,000	.445	.572	660	1140	1157	−1.7
9,000	.413	.529	610	1030	1015	1.5
8,000	.361	.465	536	896	872	2.4
7,000	.298	.389	448	748	729	1.9
6,000	.234	.311	359	599	586	1.3
5,000	.181	.243	281	461	443	1.8
4,000	.105	.150	173	293	300	− .7
3,000	.044	.068	078	138	158	−2.0
2,000	0.000	0.000	0.00000	0.00000	0.00015	−1.5
PALISADE DIABASE.						
$\Delta L_0 = 0.80$ $V = 20{,}082$ $Q = -0.05$ $10^4 a = 0.6$ $10^6 b = 1.472$						
7,840	0.350	0.442	0.00511	0.00865	0.00872	− .7
6,860	.299	.381	440	734	728	.6
5,880	.231	.300	347	582	583	− .1
4,900	.175	.230	265	441	439	.2
3,920	.116	.157	181	299	295	.4
2,940	.060	.081	093	152	150	.2
1,960	0.000	0.000	0.00000	0.00000	0.00006	− .6
BASALT.						
$\Delta L_0 = 0.07$ $V = 20{,}097$ $Q = -0.05$ $10^4 a = 0.8$ $10^6 b = 1.895$ $10^{10} c = 0.214$						
12,000	0.922	0.939	0.01083	0.01683	0.01689	− .6
11,000	.858	.873	1007	1547	1540	.7
10,000	.778	.791	912	1392	1387	.5
9,000	.696	.707	816	1236	1230	.6
8,000	.591	.599	691	1051	1068	−1.7
7,000	.509	.515	594	894	903	− .9
6,000	.423	.428	494	734	732	.2
5,000	.331	.335	386	566	557	.9
4,000	.225	.227	256	376	378	− .2
3,000	.123	.125	144	204	196	.8
2,000	0.000	0.000	0.00000	0.00000	0.00008	− .8

TABLE IV.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_s)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
BASALT.						
$\Delta L_0 = 0.865$ $V = 20{,}097$ $Q = 0.05$ $10^4a = 1.0$ $10^6b = 1.852$ $10^{10}c = 0.196$						
7,840	0.471	0.571	0.00659	0.01013	0.01031	−1.8
6,860	.423	.509	588	882	871	1.1
5,880	.341	.415	479	714	706	.8
4,900	.258	.318	367	543	538	.5
3,920	.170	.214	247	365	366	− .1
2,940	.093	.116	134	193	190	.3
1,960	0.000	0.000	0.00000	0.00000	0.00010	−1.0
WESTERLY GRANITE.						
$\Delta L_0 = -0.47$ $V = 19{,}996$ $Q = -0.58$ $10^4a = 1.0$ $10^6b = 2.168$ $10^{10}c = 0.198$						
12,000	1.346	1.202	0.01393	0.01993	0.01980	1.3
11,000	1.216	1.078	1249	1789	1801	−1.2
10,000	1.108	0.976	1130	1610	1618	− .8
9,000	0.997	.875	1014	1434	1440	− .6
8,000	.870	.759	879	1239	1239	.0
7,000	.742	.642	743	1043	1044	− .1
6,000	.603	.517	600	840	845	− .5
5,000	.479	.409	474	654	643	1.1
4,000	.326	.275	318	438	436	.2
3,000	.176	.148	172	232	225	.7
2,000	0.000	0.000	0.00000	0.00000	0.00010	−1.0
WESTERLY GRANITE.						
$\Delta L_0 = 1.57$ $V = 19{,}996$ $Q = -0.58$ $10^4a = 0.9$ $10^6b = 2.064$ $10^{10}c = 0.139$						
7,840	0.573	0.700	0.00810	0.01164	0.01175	−1.1
6,860	.488	.598	693	987	989	− .2
5,880	.409	.502	581	816	797	1.9
4,900	.297	.372	431	607	604	.3
3,920	.188	.243	281	399	408	− .9
2,940	.110	.139	161	220	210	1.0
1,960	0.000	0.000	0.00000	0.00000	0.00009	− .9
STONE MOUNTAIN GRANITE.						
$\Delta L_0 = 0.67$ $V = 20{,}068$ $Q = -0.18$ $10^4a = 1.0$ $10^6b = 2.066$ $10^{10}c = 0.152$						
7,840	0.636	0.700	0.00809	0.01163	0.01173	−1.0
6,860	.550	.607	701	995	987	.8
5,880	.433	.480	554	789	796	− .7
4,900	.343	.381	440	616	604	1.2
3,920	.233	.260	301	419	410	.9
2,940	.117	.131	151	210	211	− .1
1,960	0.000	0.000	0.00000	0.00000	0.00010	−1.0
WASHINGTON GRANITE.						
$\Delta L_0 = -1.165$ $V = 20{,}021$ $Q = -0.44$ $10^4a = 1.5$ $10^6b = 2.240$ $10^{10}c = 0.248$						
7,840	0.936	0.760	0.00880	0.01234	0.01247	−1.3
6,860	.812	.655	758	1052	1053	− .1
5,880	.673	.538	624	859	855	.4
4,900	.540	.430	497	673	652	2.1
3,920	.365	.284	329	447	445	.2
2,940	.195	.152	176	235	232	.3
1,960	0.000	0.000	0.00000	0.00000	0.00015	−1.5

TABLE IV.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_r)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			OBSIDIAN.			
$\Delta L_0 = 0.51$ $V = 20{,}030$ $Q = -0.44$ $10^4 a = -3.9$ $10^6 b = 2.861$						
12,000	1.897	1.940	0.02243	0.02843	0.02822	2.1
11,000	1.691	1.728	1998	2538	2536	− .2
10,000	1.485	1.517	1754	2234	2250	−1.6
9,000	1.307	1.344	1554	1974	1963	1.1
8,000	1.124	1.147	1326	1686	1677	.9
7,000	0.921	0.939	1086	1386	1391	− .5
6,000	.713	.726	841	1081	1105	−2.4
5,000	.536	.546	632	812	819	− .7
4,000	.320	.326	378	498	533	−3.5
3,000	.166	.170	197	257	247	1.0
2,000	0.000	0.000	0.00000	0.00000	−0.00039	3.9
			OBSIDIAN.			
$\Delta L_0 = 0.05$ $V = 20{,}030$ $Q = -0.44$ $10^4 a = 0.5$ $10^6 b = 2.857$						
7,840	1.190	1.158	0.01327	0.01681	0.01686	− .5
6,860	0.995	0.966	1117	1411	1405	.6
5,880	.799	.772	892	1127	1125	.2
4,900	.598	.575	665	841	845	− .4
3,920	.402	.384	444	562	565	− .3
2,940	.211	.201	233	292	285	.7
1,960	0.000	0.000	0.00000	0.00000	0.00005	− .5
			SERPENTINE.			
$\Delta L_0 = -0.47$ $V = 20{,}122$ $Q = 0.08$ $10^4 a = 0.9$ $10^6 b = 1.819$ $10^{10} c = 0.301$						
12,000	0.860	0.808	0.00930	0.01530	0.01527	.3
11,000	.809	.760	875	1415	1402	1.3
10,000	.721	.673	775	1255	1271	−1.6
9,000	.662	.617	712	1132	1135	− .3
8,000	.586	.542	624	984	992	− .8
7,000	.509	.473	546	846	843	.3
6,000	.423	.391	451	691	688	.3
5,000	.336	.309	356	536	528	.8
4,000	.234	.214	246	366	361	.5
3,000	.124	.113	130	190	188	.2
2,000	0.000	0.000	0.00000	0.00000	0.00009	− .9
			SERPENTINE.			
$\Delta L_0 = 1.08$ $V = 20{,}122$ $Q = 0.08$ $10^4 a = 0.4$ $10^6 b = 1.675$ $10^{10} c = 0.124$						
7,840	0.363	0.502	0.00579	0.00933	0.00946	−1.3
6,860	.317	.440	507	801	795	.6
5,880	.258	.362	418	653	642	1.1
4,900	.185	.269	310	486	486	.0
3,920	.119	.181	209	327	327	.0
2,940	.066	.092	106	165	167	− .2
1,960	0.000	0.000	0.00000	0.00000	0.00004	− .4

TABLE IV.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r - \Delta V_s/V_s)$	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			PLATE GLASS.			
$\Delta L_0 = 0.49$ $V = 20{,}148$ $Q = 0.12$ $10^4 a = -0.1$ $10^6 b = 2.232$ $10^{10} c = 0.124$						
12,000	1.178	1.292	0.01487	0.02087	0.02107	−2.0
11,000	1.097	1.203	1385	1925	1907	1.8
10,000	0.968	1.066	1227	1707	1705	.2
9,000	.861	0.949	1092	1512	1500	1.2
8,000	.737	.816	939	1299	1294	.5
7,000	.601	.669	770	1070	1084	−1.4
6,000	.477	.533	614	854	872	−1.8
5,000	.378	.423	488	668	658	1.0
4,000	.237	.269	310	430	440	−1.0
3,000	.129	.147	169	229	221	.8
2,000	0.000	0.000	0.00000	0.00000	−0.00001	.1
			PLATE GLASS.			
$\Delta L_0 = 0.545$ $V = 20{,}148$ $Q = 0.12$ $10^4 a = 0.4$ $10^6 b = 2.215$ $10^{10} c = 0.054$						
7,840	0.718	0.803	0.00926	0.01280	0.01288	− .8
6,860	.615	.688	793	1087	1077	1.0
5,880	.484	.546	630	865	865	.0
4,900	.361	.410	472	648	651	− .3
3,920	.241	.276	318	436	436	.0
2,940	.124	.143	165	224	221	.3
1,960	0.000	0.000	0.00000	0.00000	0.00004	− .4
			CAST IRON.			
$\Delta L_0 = 0.20$ $V = 20{,}008$ $Q = -0.24$ $10^4 a = -0.6$ $10^6 b = 0.884$ $10^{10} c = 0.082$						
12,000	0.174	0.172	0.00199	0.00799	0.00796	.3
11,000	.150	.148	171	711	723	−1.2
10,000	.151	.150	173	653	649	.4
9,000	.139	.137	157	577	573	.4
8,000	.129	.127	147	507	495	1.2
7,000	.099	.097	112	412	416	− .4
6,000	.090	.089	103	343	334	.9
5,000	.050	.048	56	236	252	−1.6
4,000	.051	.049	57	177	168	.9
3,000	.012	.011	13	73	82	− .9
2,000	0.000	0.000	0.00000	0.00000	−0.00006	.6
			CAST IRON.			
$\Delta L_0 = 0.49$ $V = 20{,}008$ $Q = -0.24$ $10^4 a = 0.2$ $10^6 b = 0.889$ $10^{10} c = 0.096$						
12,000	0.117	0.162	0.00191	0.00791	0.00795	− .4
10,000	.117	.156	184	664	652	1.2
8,000	.080	.113	133	493	501	− .8
6,000	.062	.084	99	339	342	− .5
4,000	.038	.052	61	181	176	.5
3,000	.020	.027	32	92	90	.2
2,000	0.000	0.000	0.00000	0.00000	0.00002	− .2

sponge is composed, and is identical with the volume-change at the same pressure of a compacted piece of the same material. This corresponds to the inner compressibility. In the second case, however, the compressibility would be much greater; besides the change of volume of the fibres themselves there would be an additional effect due to the closing in of the pores. The compressibility in this case may be called the "*outer*" *compressibility*, which then refers to the total change in the volume as measured from the ordinary boundaries of the solid when pressure is exerted wholly from the outside and not at all through the pores of the material.[29] It is the outer compressibility which determines the volume-changes of rocks under pressure due to the weight of the overlying crust,[30] and which is comparable with the elastic constants obtained by indirect methods, *e.g.*, by loading a column and measuring the longitudinal contraction and the lateral extension.

The results listed in Table IV, yielding as they do merely the inner compressibility, are therefore not sufficient entirely to determine the effect of pressure on the volume of rocks. It would be difficult to predict in advance whether the effect of porosity would be large or small; measurements, under hydrostatic pressure, of the outer compressibility are required for the substances which are known to be porous. In order to accomplish this a new set of determinations was carried out on the rock cylinders after they had been enclosed in a jacket of thin sheet tin.

The method of enclosing the rocks was as follows: A close-fitting cylinder of pure tin,[31] about 0.1 mm. in thickness and about 2 mm. longer than the rock cylinders, was pushed over the rock and its ends turned over on to discs of tin which covered the ends of the rock. The rock with its covering was then heated in an oven to about 150° and the ends sealed by dipping momentarily into molten

[29] The difference between the inner and the outer compressibility is of course dependent only on the *open* pores; the *closed* pores would give an effect common to both methods of testing.

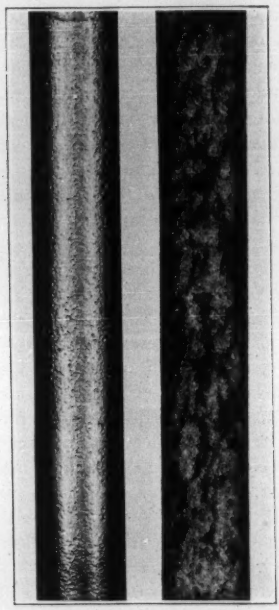
[30] We might distinguish a third kind of compressibility for porous solids in terms of the volume-change when the solid is enclosed in a thin envelope, but has its pores filled with liquid. The value of this compressibility would depend on the properties of the liquid, and would lie between the values for the inner and the outer compressibility.

[31] Commercial tooth-paste tubes are very convenient for this purpose. To make them the correct size they can be enlarged a little by stretching them over a steel rod slightly tapered at one end.

solder which consisted of 37 per cent. lead and 63 per cent. tin and which was maintained at a temperature of about 215°.

Some difficulty was experienced at first in getting the covering sufficiently leak-proof so that kerosene at 12,000 megabars could not penetrate into the rock; but with care and patience it is possi-

FIG. 4.



(*a*) (*b*)

Photograph of specimen of one rock (Washington granite), (*a*) covered with thin tin jacket, and (*b*) with the covering removed. (Natural size.)

ble to cover the rocks with a jacket which will keep out the kerosene and which at the same time is thin enough so that it can maintain only a small pressure difference, *i.e.*, so that the pressure is only a few megabars more than the pressure within the jacket. In order to ascertain whether or not the enclosure had held back the kerosene, the specimens were weighed before and after exposing them to pressure. If the weight was found to have increased, the

tin covering was taken off, the kerosene removed by heating in an oven and the cylinder enclosed again in a fresh tube. The appearance of cylinders after being subjected to pressure also served as an indication of whether or not the enclosure had held. If it had leaked, the surface of the tin usually remained smooth; but if there had been no leak, the surface took on a characteristic dimpled appearance—the tin had been forced so hard against the rock that it yielded a perfect replica in bright tin of the surface of the rock, the imperfections of which are more easily seen when copied in tin. (Fig. 4.)

In Table V are presented the results of the measurements on the enclosed rocks. The arrangement and symbols are similar to those of the preceding tables. V_{Sn} is the volume of the tin enclosure, V' the volume of the rock, and V the total volume of the enclosed specimen—all at $P=0$. The values for the volume-change at a given pressure as shown in column 4 of this table are determined by the aid of equation (2) in which V_{Sn} takes the place of V_{Cu}. From these volume-changes, as before, are calculated the coefficients, a, b, and c of equation (3), and finally the compressibility at the various pressures.

The change of volume caused by hydrostatic pressure is illustrated by Fig. 5, which shows graphically the results for two of the enclosed rocks. The decrease of volume per unit volume, $-\Delta V/V$, is plotted as ordinate, and the pressure, P, as abscissa. On a small scale, the lines appear very nearly straight, but the curvature for these two rocks actually is significant in amount, although much less than would be observed if the graph were extended to very low pressures—say two or three hundred megabars.

All of the results for rocks (and also for plate glass and cast iron) are summarized in Table VI. Four of the materials, namely, obsidian, serpentine, plate glass, and cast iron, are not porous and are absolutely impervious to kerosene under pressure; they were therefore not enclosed. The remaining eight rocks all show an appreciable porosity, an approximate value for which is given above in connection with the description of the materials. These rocks were investigated in the enclosed as well as in the unenclosed condition, except the gabbro, which was compressed only in the enclosed condition.

TABLE V.

Experimental Results for Enclosed Rocks. Volume of Reference Steel 20,054 (18,516 for Marble, and 15,477 for Gabbro).

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			MARBLE.		
$\Delta L_0 = 0.69$	$V = 18,528$	$V' = 17,886$	$V_{Sn} = 642$	$Q = -0.09$	$10^4 a = 0.0$ $10^6 b = 1.387$
12,000	0.560	0.668	0.01392	0.01387	.5
10,000	.436	.529	1106	1109	− .3
8,000	.316	.392	824	832	− .8
6,000	.212	.270	560	555	.5
4,000	.108	.140	298	277	.9
3,000	.044	.062	133	139	− .6
2,000	0	0	0	0	.0
1,000	—0.087	−0.112	—0.00196	—0.00139	−5.7
			SUDBURY DIABASE.		
$\Delta L_0 = 0.10$	$V = 20,158$	$V' = 19,492$	$V_{Sn} = 666$	$Q = 0.34$	$10^4 a = 0.1$ $10^6 b = 1.358$
$10^{10} c = 0.095$					
12,000	0.505	0.582	0.01266	0.01264	.2
10,000	.411	.478	1026	1027	− .1
8,000	.311	.366	783	782	.1
6,000	.211	.253	529	529	.0
4,000	.108	.131	270	269	.1
3,000	.056	.069	138	136	.2
2,000	0	0	0	1	− .1
1,000	−0.048	−0.056	−0.00124	−0.00136	1.2
			PALISADE DIABASE.		
$\Delta L_0 = 0.01$	$V = 20,115$	$V' = 19,361$	$V_{Sn} = 754$	$Q = 0.12$	$10^4 a = 1.0$ $10^6 b = 1.811$
$10^{10} c = 0.322$					
12,000	0.746	0.778	0.01505	0.01499	.6
10,000	.634	.659	1248	1253	− .5
8,000	.502	.522	969	981	−1.2
6,000	.371	.385	691	683	.8
4,000	.203	.211	368	359	.9
3,000	.110	.114	194	188	.6
2,000	0	0	0	10	−1.0
1,000	—0.106	—0.111	—0.00200	—0.00174	−2.6
			GABBRO.		
$\Delta L_0 = 0.57$	$V = 15,489$	$V' = 14,924$	$V_{Sn} = 565$	$Q = -0.03$	$10^4 a = -0.3$ $10^6 b = 1.347$
$10^{10} c = 0.123$					
12,000	0.289	0.383	0.01217	0.01221	− .4
10,000	.246	.328	1008	996	1.2
8,000	.176	.243	750	761	−1.1
6,000	.125	.176	522	516	.6
4,000	.059	.087	260	261	− .1
3,000	.027	.042	128	130	− .2
2,000	0	0	0	3	.3
1,000	−0.054	−0.076	−0.00184	−0.00139	−4.5

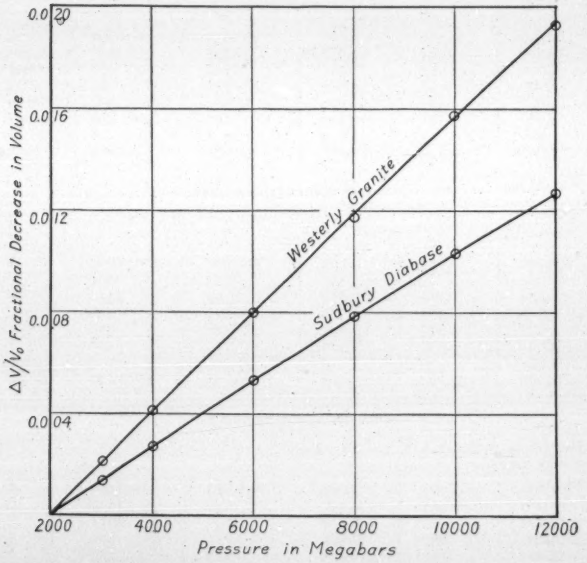
TABLE V.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			BASALT.		
$\Delta L_0 = 1.09$ $V = 19{,}983$ $V' = 19{,}244$ $V_{Sn} = 739$ $Q = -0.52$ $10^4a = 1.2$ $10b^6 = 2.431$ $10^{10}c = 0.455$					
12,000	1.122	1.226	0.01988	0.01988	.0
10,000	.953	1.051	1673	1666	.7
8,000	.744	.822	1295	1307	−1.2
6,000	.530	.588	910	912	− .2
4,000	.289	.321	486	480	.6
3,000	.160	.178	264	251	1.3
2,000	0	0	0	12	−1.2
1,000	−0.176	−0.199	−0.00288	−0.00236	−5.2
			WESTERLY GRANITE.		
$\Delta L_0 = 1.01$ $V = 19{,}295$ $V' = 18{,}527$ $V_{Sn} = 768$ $Q = 3.47$ $10^4a = 0.2$ $10^6b = 2.010$ $10^{10}c = 0.076$					
12,000	1.405	1.060	0.01932	0.01936	− .4
10,000	1.181	.873	1577	1562	1.5
8,000	.903	.641	1164	1181	−1.7
6,000	.646	.444	797	794	.3
4,000	.349	.229	407	401	.6
3,000	.181	.113	201	202	− .1
2,000	0	0	0	2	− .2
1,000	−0.245	−0.148	−0.00250	−0.00200	−5.0
			STONE MOUNTAIN GRANITE.		
$\Delta L_0 = 0.64$ $V = 20{,}131$ $V' = 19{,}530$ $V_{Sn} = 601$ $Q = 0.06$ $10^4a = 0.5$ $10^6b = 1.975$ $10^{10}c = 0.093$					
12,000	0.964	1.093	0.01886	0.01887	− .1
10,000	.785	.895	1533	1526	.7
8,000	.580	.670	1148	1157	− .9
6,000	.390	.456	776	780	− .4
4,000	.206	.244	408	396	1.2
3,000	.099	.120	201	202	− .1
2,000	0	0	0	5	− .5
1,000	−0.123	−0.152	−0.00240	−0.00193	−4.7
			WASHINGTON GRANITE.		
$\Delta L_0 = 0.84$ $V = 19{,}383$ $V' = 18{,}741$ $V_{Sn} = 642$ $Q = -3.10$ $10^4a = 0.2$ $10^6b = 2.323$ $10^{10}c = 0.345$					
12,000	1.424	1.107	0.01980	0.01980	.0
10,000	1.214	.931	1639	1639	.0
8,000	.971	.731	1271	1271	.0
6,000	.692	.507	871	876	− .5
4,000	.383	.273	462	453	.9
3,000	.197	.135	228	231	.3
2,000	0	0	0	2	− .2
1,000	−0.222	−0.133	−0.00225	−0.00233	.8

TABLE V.—*Continued.*

Pressure (Megabars).	ΔL	ΔL (corr.)	$-(\Delta V_r/V_r)$ (obs.)	$-(\Delta V_r/V_r)$ (calc.)	Diff. $\times 10^4$
			WASHINGTON GRANITE.		
$\Delta L_0 = 1.01$ $V = 19{,}383$ $V' = 18{,}741$ $V_{Sn} = 642$ $Q = -3.10$ $10^4 a = -0.1$ $10^6 b = 2.255$ $10^{10} c = -0.293$					
12,000	1.378	1.090	0.01959	0.01961	− .2
10,000	1.179	.921	1627	1616	1.1
8,000	.925	.705	1238	1247	− .9
6,000	.664	.494	855	854	.1
4,000	.355	.254	436	438	− .2
3,000	.186	.129	221	222	− .1
2,000	0	0	0	1	.1
1,000	−0.205	−0.123	−0.00212	−0.00229	1.7

FIG. 5.



The decrease in volume of two of the materials investigated, at pressures from 2000 to 12,000 megabars. The observed points lie on smooth curves, which on the small scale of the diagram appear to be straight lines. Actually the curvature is appreciable.

TABLE VI.
*Summary of Results for Rocks.**

Material.	Coeffs. of Eq. (4)		Compressibility, $\beta \times 10^6$		
	$b \times 10^6$	$c \times 10^{10}$	at 0	at 2000	at 10,000
Marble unenclosed	1.411	.....	....	1.41	1.41
Marble enclosed	1.387	.....	....	**1.39**	**1.39**
Sudbury diabase unenclosed	1.255	.....	....	1.26	1.26
Sudbury diabase enclosed	1.358	0.095	....	**1.36**	**1.21**
Palisade diabase unenclosed	1.442	.....	....	1.44	1.44
Palisade diabase enclosed	1.811	0.322	....	**1.81**	**1.30**
Basalt unenclosed	1.881	0.210	....	1.88	1.55
Basalt enclosed	2.431	0.455	....	**2.43**	**1.70**
Gabbro enclosed	1.347	0.123	....	**1.35**	**1.15**
Westerly granite unenclosed	2.133	0.186	....	2.13	1.84
Westerly granite enclosed	2.010	0.076	....	**2.01**	**1.89**
Stone Mt. granite unenclosed	2.066	0.152	....	2.07	1.82
Stone Mt. granite enclosed	1.975	0.093	....	**1.98**	**1.83**
Washington granite unenclosed	2.240	0.248	....	2.24	1.84
Washington granite enclosed	2.289	0.319	....	**2.29**	**1.78**
Obsidian	2.860	.....	**2.86**	**2.86**	**2.86**
Serpentine (Talc Schist)	1.804	0.266	....	**1.80**	**1.38**
Plate glass	2.226	0.110	**2.27**	**2.23**	**2.05**
(Cast iron)	0.887	0.089	....	**0.89**	**0.76**

*The values for the enclosed rocks more truly represent the compressibilities of the materials.

COMPARISON WITH THE RESULTS OF EARLIER INVESTIGATORS.

Minerals.—A number of measurements on the compressibility of minerals have been made by Voigt[32] and by Madelung and Fuchs.[33] Voigt determined the elastic constants by bending and by twisting the crystals and measuring the deformation produced by a given stress. Madelung and Fuchs measured the compressibility under hydrostatic pressure, using a piezometer and working at a mean pressure of 125 megabars. For the sake of comparison some of the results of Madelung and Fuchs and all of Voigt's results are listed in Table VII. Voigt's values have been recalculated to correspond with our unit of pressure, the megabar. The results of Madelung and Fuchs with few exceptions are in good agreement with our measurements. Their value for silica glass is noticeably lower, and that for orthoclase is somewhat lower than the corresponding values obtained by us, but in the remaining cases where comparison is possible the agreement is excellent. Voigt's

[32] W. Voigt, *Ann. Physik*, 1887, **31**, 474–501, 701–24; 1888, **34**, 981–1028, **35**, 642–61; 1890, **39**, 412–31, **41**, 712–24.

[33] E. Madelung and R. Fuchs, *Ann. Physik*, 1921, **45**, 289–309.

figures agree well in some cases, but for others they are much too high. But taking into consideration the difficulty of measuring the compressibility of crystals by an indirect method, the results of Voigt are, on the whole, surprisingly near the truth. The compressibilities of sylvite and halite have also been measured by Richards and Jones.[34] Their values, recalculated on the basis of the compressibility of mercury being 3.96×10^{-6} per megabar, are as follows: 5.20×10^{-6} for fused potassium chloride,

TABLE VII.

Some Determinations of the Compressibility of Minerals by Other Investigators.

Mineral.	Voigt $\beta \times 10^6$ (Pressure very low).	Madelung and Fuchs $\beta \times 10^6$ $P = 125$	A. & W. $\beta \times 10^6$ $P = 0$
Fluorite	1.18	1.24	....
Pyrite	1.13	0.71	0.71
Halite	4.13	4.13	4.12
Sylvin	7.37	5.62	....
Calcite	1.54	1.34	1.39
Tourmalin	1.15	0.84	....
Beryl	0.74	0.57	....
Quartz	2.65	2.66	2.70
Barite	1.92	1.76	....
Topaz	0.60	....	....
Magnetite	....	0.54	....
Corundum	....	0.41	....
Orthoclase	....	1.74	1.92
Silica glass	....	2.69	3.10
Magnesium oxide	....	0.71	....

4.28×10^{-6} for fused sodium chloride, and 4.50×10^{-6} for natural halite.

In connection with the compressibility of diamond it is interesting to note the value calculated by Thomson on the basis of his theory of the structure of solids.[35] For the compressibility of diamond the value obtained is 0.18×10^{-6}, in exact agreement with the mean of our measurements as given in Table III.

Rocks.—By measuring the longitudinal compression and the lateral extension under a unidirectional thrust, F. D. Adams and E. G. Coker[36] determined the compressibilities of a number of rocks and minerals. Their results, converted to the units used in

[34] T. W. Richards and Grinnell Jones, *J. Am. Chem. Soc.*, 1909, **31**, 158–91.

[35] J. J. Thomson, *Phil. Mag*, 1922, **43**, 721–57.

[36] Carnegie Inst. Pub. No. 46.

this paper, are given in Table VIII. It is noticeable that, in the cases of substances used both by them and by us, there are considerable discrepancies except for plate glass and diabase. For instance, their values for cast iron and Westerly granite are 60

TABLE VIII.

Results of F. D. Adams and E. G. Coker at Low Pressures (Average about 300 Megabars); and, Where Comparison Is Possible, the Results of the Present Paper at the Lower End of Pressure Range—2000 Megabars.

Material.	Compressibility, $\beta \times 10^6$ A. & C.	Compressibility, $\beta \times 10^6$ A. & W.	Remarks.
Wrought iron	0.68	0.60	Bessemer steel standard of reference.
Cast iron	1.45	0.89	
Colorado marble	....	1.39	Identical with pure calcite (by direct method). All marbles at high pressures would give this result.
Black Belgian marble	1.74	....	
Carrara marble	2.44	....	
Vermont marble	2.72	....	
Tennessee marble	2.43	....	
Montreal limestone	2.35	....	
Baveno granite	3.15	....	
Peterhead granite	3.03	....	
Lily Lake granite	3.22	....	
Westerly granite	3.30	2.01	Sample cut from material furnished by A. & C. All granites practically same β at high pressures.
Quincy granite (1)	3.64	....	
Quincy granite (2)	3.18	....	
Stanstead granite	3.68	....	
Stone Mountain granite	....	1.98	
Washington granite	....	2.29	
Nephelite syenite	2.33	....	
New Glasgow anorthosite	1.74	....	
Mount Johnson essexite	2.15	....	
New Glasgow gabbro (1)	2.13	....	
New Glasgow gabbro (2)	1.18	1.35	Sample from A. & C.
Sudbury diabase	1.36	1.36	Sample from A. & C.
Ohio sandstone	8.00	....	
Plate glass	2.25	2.23	

per cent. higher than ours, while an agreement to better than 1 per cent. is shown for the two previously mentioned substances.

There can be no doubt that the reason lies in the fact that, for the less compact materials, the bulk compressibility at low pressures changes rather rapidly with the pressure, and our experiments

began only at pressures considerably above the highest used by the others. If Hooke's law holds, in other words if the elasticity does not change with pressure, the change of volume produced by a unidirectional thrust is one-third that due to a hydrostatic pressure of the same intensity. But for substances whose compressibility changes with pressure there is a flaw in the assumption that the addition of two other equal thrusts at right angles to the first, so as to produce a total hydrostatic pressure, will each cause a change of volume equal to that produced by the first. If Hooke's law does not hold, more especially if the elastic properties are changing *rapidly* under pressure, the successive changes of volume will diminish so that the change of volume produced by the first will be greater than one-third of the total change, and compressibilities determined in this fashion will be too high.

If these ideas are correct, careful analysis of the results should show a decided change in the decrease of volume found by Adams and Coker for each increment of thrust. We have therefore calculated [37] and tabulated these values for the substances for which most experimental results are given. At each pressure the averages of the various measurements, of lateral extension and longitudinal contraction, were taken and the relative change of volume computed. In Table IX, these are compared with the values calculated from the equations.[38] In the cases of plate glass and diabase straight lines agreed sufficiently closely with the experimental data, but the differences in the case of the other substances justify the use of curves. Therefore the equations of the most probable

[37] In comparing compressibilities as determined by the two methods, (1) direct measurement under hydrostatic pressure, and (2) indirect determination from the deformation produced by a thrust, we must choose between two procedures: (*a*) The volume-change for a given unidirectional pressure is compared with the volume-change under a hydrostatic pressure equal to one-third of the unidirectional pressure; or (*b*) three times the volume-change produced by a given thrust is compared with the volume-change due to a hydrostatic pressure of the same intensity. In other words, the results of the indirect measurements of compressibility are recalculated (*a*) by dividing the pressures by 3, or (*b*) multiplying the volume-changes by 3. We have considered it more reasonable to adopt method (*b*) in comparing our values with those of Adams and Coker. The two procedures, (*a*) and (*b*), would yield identical results if the volume-changes were strictly proportional to the pressure, but when the compressibility changes with the pressure, procedure (*a*) would lead to a greater change of compressibility with pressure.

[38] Given at foot of Table IX.

parabolas to represent the data were calculated according to the same procedure as that used with our higher pressure results, and the values from those equations tabulated beside the observed values.

The compressibilities of plate glass and Sudbury diabase, 2.23 and 1.38, respectively, as calculated from these equations, agree as well as could be expected with the values obtained by Adams and Coker from the same data but by a different method of computa-

TABLE IX.

Volume-changes Calculated from Results of Adams and Coker from Five Materials. For Plate Glass and Sudbury Diabase, the Relation of Volume-change to Pressure is Linear; for the Other Solids Curves Are Used.

	Plate glass.		Sudbury diabase.		Cast iron.		Westerly granite.		Sandstone.	
Pressure Megabars.	$3\times\frac{\Delta V}{V}\times 10^5$		$3\times\frac{\Delta V}{V}\times 10^5$		$3\times\frac{\Delta V}{V}\times 10^5$		$3\times\frac{\Delta V}{V}\times 10^5$		$3\times\frac{\Delta V}{V}\times 10^5$	
	obs.	from eqn.*	obs.	from eqn.†	obs.	from eqn.‡	obs.	from eqn.§	obs.	from eqn.¶
69	0	0	0	−2	0	−1	0	2	0	0
138	16	15	7	8	14	13	29	30	72	70
207	31	31	18	18	25	26	60	55	128	130
276	46	46	28	27	38	39	81	79	179	179
345	61	61	35	37	52	51	97	101	...	...
414	76	77	46	46	63	62	119	121	...	...
483	92	92	54	56	72	73	138	139	...	...
552	107	107	67	65	83	83	158	156	...	...
621	124	123	76	76	92	92	170	170	...	...

* Plate glass. $3\times\frac{\Delta V}{V}\times 10^6 = 2.23\,(P-69)-3.$

† Sudbury diabase. $3\times\frac{\Delta V}{V}\times 10^6 = 1.38\,(P-69)-15.$

‡ Cast iron. $3\times\frac{\Delta V}{V}\times 10^6 = 2.04\,(P-69)-6.1\,(P-69)^2\,10^{-4}-10.$

§ Westerly granite. $3\times\frac{\Delta V}{V}\times 10^6 = 4.12\,(P-69)-19.6\,(P-69)^2\,10^{-4}+20.$

¶ Sandstone. $3\times\frac{\Delta V}{V}\times 10^6 = 10.9\,(P-69)-55\,(P-69)^2\,10^{-4}.$

tion. The initial compressibilities of the other three substances come out higher, but the second term in the equation lowers the values rapidly at the higher pressures. The rate of change of the compressibility as shown by the second term is much greater than it is at higher pressures according to our results.

An equation to fit all the results, *i.e.*, those from 69 to 1200 megabars, may easily be constructed by the introduction of an exponential term. The following equations are suggested for the compressibilities:

Cast iron $\quad 10^6\beta = 0.887 - 0.178(P-2000)10^{-4} + 0.022\,e^{-0.002(P-2000)}$ (5)

Westerly granite $\quad 10^6\beta = 2.010 - 0.152(P-2000)10^{-4} + 0.006\,e^{-0.003(P-2000)}$ (6)

Above a pressure of 2000 megabars the exponential terms are neglible and these equations become simply those obtained from our high pressure results. At lower pressures the exponential term increases the value of β up to the values given by Adams and Coker's results, 2.12 and 4.46 at zero pressure. Integration shows that the effect of the exponential term is to add to the change of volume as calculated from the high pressure results a small amount (0.06 per cent. of the total volume for cast iron, and 0.08 per cent. for Westerly granite) which is due to some kind of packing effect at low pressures.

The elastic constants of rocks have also been measured by Nagaoka.[39] Specimens of the rocks were subjected to bending and to twisting, and the resultant deformations used to calculate Young's modulus E and the rigidity R. As has already been pointed out by Adams and Coker, the method used by Nagaoka when applied to rocks does not yield very satisfactory results—at least not for compressibility. In nearly every case the value of E given by Nagaoka is greater than three times R. Now, the relations between the elastic constants are such that when E/R is greater than 3, the compressibility must be negative—a situation which effectively bars any comparison between Nagaoka's results and ours.

The complete curves for the four substances, Sudbury diabase, Westerly granite, plate glass, and cast iron are plotted in Fig. 6. This diagram shows the course of the compressibility from zero to 12,000 megabars, and indicates how, for a given solid, a single curve may represent both the low-pressure results of Adams and Coker and the high-pressure results of the present paper.

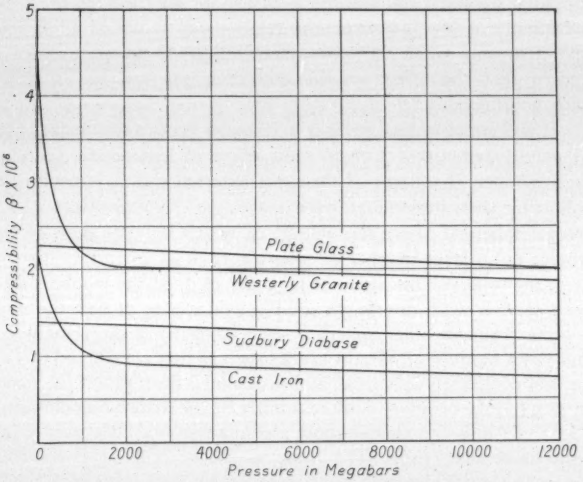
DEPENDENCE OF COMPRESSIBILITY OF ROCKS ON VARIOUS FACTORS.

Porosity and Compactness.—It would naturally be expected that the compressibility of a rock would depend to a considerable extent on its porosity. That is, the "outer" compressibility should increase with the amount of pore space; the "inner" compressibility, of course, being unaffected, at least in the case of open pores. The effect of porosity may be judged first by the differences between these two compressibilities, and second by comparing the compressibilities at high pressures with those determined at low

[39] H. Nagaoka, "Elastic Constants of Rocks and Velocity of Seismic Waves," *Phil. Mag.*, 1900, **50**, 53–68.

pressures by Adams and Coker. By referring to Table VI it may be noted that the basalt and the Palisade diabase show a much higher compressibility when enclosed in the jacket of thin sheet tin than when unenclosed. The Sudbury diabase is somewhat more compressible, while the marble and the granites show very little difference, and in some cases appear to be actually less compressible in the enclosed condition. The differences, however, for the

FIG. 6.



The compressibility of four materials from 0 to 12,000 megabars. The values at low pressures were obtained from the data of F. D. Adams and E. G. Coker.

marble and the granites are hardly more than the experimental error.

The values with the enclosed rocks are to be regarded as the more accurate, principally because the volumes of the enclosed cylinders can be determined with greater precision. The enclosed Palisade diabase, while more compressible at low pressures than the unenclosed rock, appears to be slightly less compressible at high pressures. Possibly in this case the volume-change of the unenclosed rock, which was computed as a straight line, really follows a curve, so that the compressibility at low pressures is

higher and the compressibility at high pressures is lower than the mean, 1.44×10^{-6}, recorded in the table.

The compressibility of most granites and of some basic rocks seems to be considerably higher at very low pressures than at higher pressures. Moreover, the compressibility of one rock, the basalt, is notably greater in the enclosed condition than in the unenclosed condition, even at 2000 megabars. These differences might be thought to be due to the porosity of the rocks, but although porosity doubtless is a factor in determining the compressibility at low pressures, the relationship is not at all simple, and it is only when the exact distribution of the pore space is known that the effect on the measurements can be estimated with accuracy.

It will simplify the problem to consider certain particular cases of pore-distribution. Consider the effect of hydrostatic pressure on a number of spheres of the same external radius but with the following distributions of pore space: (*a*) No cavities; (*b*) a central spherical cavity the volume of which is 1 per cent. of the whole; (*c*) a large number of spherical cavities uniformly distributed, the total volume again being 1 per cent. of the whole; (*d*) a very narrow crack dividing a solid central sphere of half the total volume from an outer spherical shell comprising the remainder; and (*e*) a number of narrow cracks such as that considered in (*d*) but not completely spherical, and distributed throughout the mass. In case (*a*) the ordinary compressibility of the material is obtained. Cases (*b*) and (*c*) should show a compression a little over 2 per cent. more than (*a*). The same would be true of pores consisting of small channels ramifying through an otherwise sound material. In case (*d*) the initial compressibility will be that of the outer spherical shell and will be between two and three times as great as that of the solid sphere; but if the crack has a volume of only a few tenths of 1 per cent. of the total, it will close up completely when the pressure reaches a few hundred megabars, and at this pressure the compressibility will increase rapidly up to the normal value for the homogeneous material. Case (*e*) would show the same characteristics as (*d*) but the change in compressibility would be more gradual.

None of the rocks has enough porosity to explain, on the basis of detached voids or channels, more than a few per cent. increase

of compressibility. The Palisade diabase with less than 0.1 per cent. porosity shows an outer compressibility at 2000 megabars distinctly higher than the inner compressibility. The granites, moreover, although containing less than 1 per cent. of pore space, show an abnormally high compressibility at very low pressures; the value at a pressure of only a few hundred megabars being nearly double that at 1000 megabars. The total volume-change when Westerly granite is exposed to a pressure of 1000 megabars is only 0.3 per cent.; yet its compressibility at this pressure has fallen off to a much lower and nearly constant value.

It appears as if there were two main factors in causing the compressibility of some rocks to have an abnormally high value at low pressures. In the first place, there is probably a sort of looseness of structure,[40] or a lack of compactness in the rock. It is as if the mineral grains fit together rather loosely, and become tightly locked only after a certain amount of pressure is applied. Pores in the sense of small channels or detached voids would have very little effect; it is with relatively broad areas separating the grains that we have to deal (*cf.* case (*e*) above). The second factor is the combination of hard and soft (or rather, more compressible and less compressible) mineral grains in the same rock. The more compressible minerals themselves show a decreasing compressibility with increase of pressure, but this effect is relatively small. The granites, which as a class exhibit a very rapid drop in compressibility for the first few hundred megabars, consist of the highly compressible quartz and mica together with the less compressible feldspars. The case of cast iron is especially striking. Although it has no porosity, its compressibility decreases from about 2×10^{-6} at very low pressures to less than 0.9×10^{-6} at 2000 megabars. Here the admixture of the very compressible graphite and the very incompressible iron carbide is probably related to the initial rapid change in compressibility. On the other hand, marble consists of a single mineral and yet shows a large drop in compressibility, coming down to 1.4×10^{-6}, the value for calcite, at less than 2000 megabars. In this case the behavior can be ascribed only to that special

[40] The word structure is here used in its broadest sense.

kind of porosity connected with a looseness of assemblage of the mineral grains.[41]

Very often the abnormally high compressibility at low pressures is associated with coarseness of grain—for example, the granites and marbles, which are comparatively coarse-grained, show a high initial compressibility. The Sudbury diabase, however, is moderately coarse-grained and does not show much of this effect, while the serpentine, which is very fine-grained, shows a considerable decrease in compressibility and this tendency to decrease persists even at high pressures. For the latter substance the behavior is doubtless due to the admixture of the less compressible calcite with the more compressible talc. Palisade diabase also shows a considerable change of compressibility at high pressures, which is difficult to explain on any basis. It is fine-grained, almost nonporous, and consists largely of two minerals, augite and plagioclase ($Ab_{40}An_{60}$) differing but little in compressibility.

About all that can be said concerning the way in which the compressibility of rocks changes with pressure is: For pressures above 2000 megabars the compressibility does not change very much; at pressures of only a few hundred megabars the compressibility is likely to be notably higher; and that the change of compressibility at low pressures, while connected with the admixture of minerals of different compressibility and with a looseness of structure existing in some coarse-grained rocks, can not with certainty be predicted in advance.

Effect of Glass.—The compressibility of a glass is probably always greater than that of a crystalline aggregate of the same chemical composition. The specific volume of the glass (with very few exceptions) is also higher, and it was thought at first that there might be a rough parallelism between volume and compressibility; but apparently no simple quantitative relation exists. Silica glass with 20 per cent. greater volume than quartz shows (at 2000 megabars) 15 per cent. greater compressibility, but on comparing the obsidian and the Westerly granite—which have nearly the same chemical composition—it may be noted that the obsidian with 10 per cent. greater volume has 40 per cent. greater compressibility. Possibly the difference in compressibility of these two

[41] The porosity and lack of compactness of the granites may, as has been suggested by various investigators, be due to the quartz-inversion, which takes place at 575° with a large change of volume.

forms of silica is small because the crystalline form itself has a comparatively high compressibility. As a rough estimate one can assume that for ordinary rocks the compressibility of the glassy material would be from one-fourth to one-third greater than that of the holocrystalline material of the same chemical composition.[42] The compressibility of two crystalline modifications of the same substance may, of course, differ, but usually not enough to destroy the utility of the suggested approximation for glasses.

Relation of Rock-compressibility to That of the Constituent Minerals.—The fact that the basic rocks have a low compressibility and consist of minerals of low compressibility, and that the acid rocks have a much higher compressibility and consist of more compressible minerals suggests that the compressibility of a rock may be the average compressibility of the constituent minerals. This expectation has been fulfilled. Except at very low pressures, where the compressibility may be abnormally high, the compressibility of a rock can be calculated with fair accuracy from the known compressibility of its minerals. In Table X are shown the results of such calculation for seven holocrystalline rocks. The second column of this table contains the observed compressibilities copied from Table VI; the third column contains the compressibilities calculated from the modes, which are given above in connection with the description of the materials used. In the fourth and fifth columns are given the corresponding values at 10,000 megabars. To calculate the compressibility, the volume-percentage

TABLE X.

Comparison of the Observed Compressibility with the Values Calculated from the Compressibility of the Constituent Minerals.

Rock.	Compressibility at 2000 megabars, $\beta \times 10^6$		Compressibility at 10,000 megabars, $\beta \times 10^6$	
	Observed.	Calculated.	Observed.	Calculated.
Westerly granite	2.01	2.09	1.89	1.86
Stone Mountain granite	1.98	2.04	1.83	1.81
Washington granite	2.29	2.06	1.78	1.85
Sudbury diabase	1.36	1.24	1.21	1.19
Palisade diabase	1.81	1.22	1.30	1.18
New Glasgow gabbro	1.35	1.08	1.15	1.06
Colorado marble	1.39	1.39	1.39	1.39

[42] The problem is complicated by the fact that in partially crystalline rocks the vitreous part seldom has the composition of the rock as a whole.

of each mineral is multiplied by its compressibility; the sum of these products is then the "calculated" compressibility as shown in Table X. The compressibilities of the plagioclases for use in these calculations were interpolated from the values for albite and anorthite as given in Table III; the compressibilities of biotite and muscovite were assumed to be the same as for phlogopite; for that of epidote was taken the value for anorthite; and for that of olivine was taken 0.93, as estimated above (p. 494).

The observed and calculated values, on the whole, show a satisfactory agreement, especially so when the difficulty of accurately determining the mode is taken into account. The correspondence at 10,000 megabars is rather better than at 2000. The three granites are very close together as might be expected from the fact that the chief factor in yielding a high compressibility is quartz, which for the three granites varies only from 20 to 32 per cent. The Palisade diabase shows the greatest discrepancy between observed and calculated compressibility, while marble shows the least, the two values in this case being identical. The discrepancy for the Palisade diabase may possibly be connected with the presence of a saussuritized feldspar (included with plagioclase in the estimate of the mode). The rock is evidently not perfectly fresh, and the lessened compactness due to slight alteration would probably account for a higher compressibility especially at low pressures. A calculation for the basalt was not made because this rock contains a considerable amount of glass, the compressibility and composition of which is not known. The relatively high compressibility of the basalt, however, is another indication of the greater compressibility of a glass as compared with crystals of the same composition. The average discrepancy between observed and calculated compressibilities in Table X is 7 per cent., and neglecting the Palisade diabase the average is 5 per cent.

We thus come to the important conclusion that at moderately high pressures the compressibility of a fresh holocrystalline rock is an additive function of the compressibility of its minerals, provided that the pressure is not too low; in spite of considerable porosity, rocks at about 2000 megabars or more reach a compressibility which can be calculated from the mineral composition. But for certain rocks, notably the granites and probably also the other acid plutonic rocks, the compressibility below 1000 or 2000 megabars may be abnormally high. Thus, the compressibility of granite

may be above 4×10^{-6} at a pressure of 100 megabars—over twice as great as the compressibility at high pressures. On the other hand there are probably many rocks which maintain what we may call their normal compressibility down to very low pressures. An excellent example is the Sudbury diabase, which is a compact rock of low porosity. The compressibility, as shown by the measurements of Adams and Coker, is 1.36×10^{-6} at very low pressures, identical with our figure at 2000 megabars. For one specimen of the New Glasgow gabbro Adams obtained a lower value, and for another specimen a higher value, than ours. Here the relations are somewhat obscured by the obvious variability of the material, and indicate again the difficulty of predicting the compressibility of rocks at *very low* pressures.

The fact that the compressibility of rock masses can be calculated from the known compressibility of the constituent minerals is especially useful for the study of phenomena pertaining to those parts of the earth's crust which are not too near the surface. In the next section we proceed to make use of the relation between mineral- and rock-compressibility in listing certain elastic properties of a number of typical rocks.

ELASTIC CONSTANTS OF TYPICAL ROCKS.

The compressibilities of a number of types of plutonic igneous rocks have been calculated from the assumed mineral content, and the results are recorded in Table XI. The kind and amount of the minerals for each rock, as shown in the table, have been chosen so as to characterize the rock in terms of the essential constituents. The order of arrangement is that of increasing basicity, or decreasing percentage of silica. In order to round out the series, we include dunite, a massive igneous rock composed wholly of olivine; pallasite,[43] a mixture of iron and olivine composing certain meteorites although not found as a terrestrial surface rock; and also metallic iron. Siderite, a name used for meteorites composed wholly of metallic iron with a little nickel, is not to be confused with the terrestrial mineral siderite—ferrous carbonate.

The compressibility at two pressures (2000 and 10,000 megabars) is calculated from the compressibility of the constituent minerals according to the method already explained in connection

[43] Pallasite may be considered to consist, as a rough average, of about equal parts of olivine and metallic iron.

TABLE XI.

Compressibility and Rigidity of Typical Plutonic Rocks at High Pressures; and the Velocity of Earthquake Waves as Determined by the Elastic Constants. The Compressibility is Calculated from the Compressibility of the Constituent Minerals, and the Rigidity is Computed by Assuming that at High Pressures Poisson's Ratio is 0.27.

Rock.	Suggested Average Composition (by Volume).											Pressure Megabars.	Elastic Constants.			Density. ρ.	Velocity of Seismic Waves Km./sec.		Rock.
	Quartz.	Orthoclase.	Andesine. Ab_3An_2	Labradorite. Ab_1An_1	Biotite.	Amphibole.	Augite.	Hypersthene.	Olivine.	Metallic Iron.			Compressibility. $\beta \times 10^6$	Bulk Modulus. $K \times 10^{-6}$	Rigidity. $R \times 10^{-6}$		Longitudinal. (P).	Transverse. (S).	
Granite	30	65	..	..	5	..	..	..	..	..		2,000	2.12	0.47	0.26	2.61	5.6	3.1	Granite
												10,000	1.88	0.53	0.29	2.66	5.9	3.3	
Granodiorite	11	20	52	..	10	7	..	..	..	..		2,000	1.83	0.55	0.30	2.69	5.9	3.3	Granodiorite
												10.000	1.66	0.60	0.33	2.73	6.2	3.5	
Syenite	..	90	..	..	5	5	..	..	..	..		2,000	1.87	0.54	0.29	2.61	5.9	3.3	Syenite
												10,000	1.68	0.60	0.32	2.66	6.2	3.5	
Diorite	..	..	80	..	5	15	..	..	..	..		2,000	1.62	0.62	0.34	2.74	6.2	3.5	Diorite
												10,000	1.49	0.67	0.36	2.78	6.4	3.6	
Gabbro	..	..	..	50	..	..	40	10	..	..		2,000	1.20	0.83	0.45	3.05	6.9	3.9	Gabbro
												10.000	1.17	0.85	0.46	3.08	6.9	3.9	
Pyroxenite	..	..	..	..	..	..	50	50	..	..		2,000	1.03	0.97	0.53	3.40	7.0	3.9	Pyroxenite
												10,000				3.44			
Peridotite	..	..	..	..	..	..	..	50	50	..		2,000	0.97	1.03	0.56	3.40	7.2	4.0	Peridotite
												10,000				3.44			
Dunite	..	..	..	..	..	..	..	..	100	..		2,000	0.93	1.08	0.59	3.38	7.4	4.1	Dunite
												10,000				3.41			
Pallasite	..	..	..	..	..	..	..	..	50	50		2,000	0.77	1.30	0.71	5.65	6.3	3.5	Pallasite
												10,000				5.69			
Siderite	..	..	..	..	..	..	..	..	..	100		2,000	0.60	1.67	0.91	7.9	6.0	3.4	Siderite
												10,000							

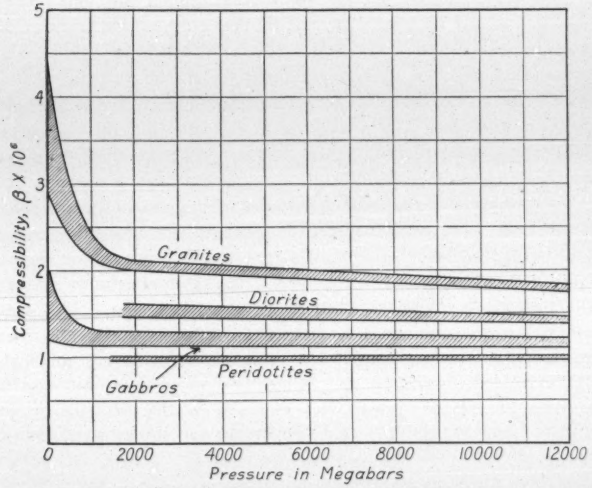
with Table X. In the column adjoining the compressibilities are the corresponding values of the bulk modulus, which is obtained by taking the reciprocal of the compressibility. This quantity is of the same dimensions as pressure, *e.g.*, megabars, and is denoted by the letter K.

From a study of this table it may be seen that the compressibility decreases (and the bulk modulus, of course, increases) as we go down the list—except for syenite, which falls out of line. The more basic a rock, in general the less compressible it is. This may be considered a consequence of the inherently great compressibility of silica, even in combination, and also of the alkalies, which, as is well known, are more abundant in the acid rocks. Syenite, however, is an exception to the rule that the compressibility goes up as the silica goes down. Although the syenite has less silica than the granodiorite, its compressibility is greater. But it must be recalled that when rocks are arranged in order of basicity, syenite is to be considered a member of a branching series, and not a member of the main series in which quartz and orthoclase decrease together; in passing from granodiorite to syenite, the orthoclase increases while the quartz decreases. The same anomaly for syenite may be noticed in the case of the densities, which have been calculated from the densities of the minerals, and recorded in column 16. There is a progressive increase of density, except for syenite with its lower density than the granodiorite which it follows. If, however, we had chosen a granodiorite of slightly different composition, namely, quartz, 20; orthoclase, 25; andesine, 40; biotite, 10; and amphibole, 5, the compressibility of the granodiorite would have been sufficiently higher so that in Table XI the syenite would not have been an exception to the general rule of decreasing compressibility, as the rocks become more basic. But the densities of the granodiorites change so little for any reasonable range of composition that with respect to density syenite would still fall out of line.

It may be noted that the compressibility of the more acid rocks is a little less than that of plate glass ($\beta = 2.2 \times 10^{-6}$), while that of the ultra-basic rocks (*e.g.*, a peridotite) is only about 50 per cent. greater than that of steel. The compressibilities of all ordinary rocks at moderate pressures lie between that of silver and that of plate glass.

Attention is called to the fact that although the compressibility of the more basic rocks appears to be constant throughout the pressure range from 2000 to 12,000 megabars, this is merely because the change in compressibility is too small to measure by the ordinary methods. With greater refinements of measurement, or with a greater range of pressure, there would undoubtedly be for all

FIG. 7.



The compressibility of four types of igneous rocks as a function of pressure. The width of the shaded areas indicates the variation which may be expected for ordinary rocks within the given class. At high pressures the variation is almost wholly due to the variation in composition; at low pressures, however, the range of variation is much larger and is dependent on the compactness of the rock.

substances a small but significant increase of bulk modulus, or decrease of compressibility, as the pressure is increased.

The compressibilities of four important rock types, granite, diorite, gabbro, and pyroxenite, are indicated in Fig. 7. The width of the shaded areas is intended to suggest the approximate variation in compressibility which may be expected to occur in the more common rocks of the given type. The assumed range of composition—which of course does not cover *all* rocks of the given type—is as follows: For the granites, from 30 to 15 per cent. quartz,

the biotite remaining constant at 5 per cent., and the remainder being orthoclase; for the diorites, from the composition given in Table XI to 40 per cent. andesine and 60 per cent. of amphibole; for the gabbros, from 25 to 75 per cent. labradorite, the remainder being augite; and for the pyroxenites, from 25 to 75 per cent. hypersthene, the remainder being olivine. The results of Adams and Coker, for six different granites, provide the basis for the estimate of the range of variation for the granites, at low pressures. The corresponding variation for the gabbros at low pressures is admittedly a very rough estimate representing mainly the opinion of the authors. The graphs for diorite and pyroxenite are not extended below 2000 megabars. It should be borne in mind that the variation at high pressures is almost entirely dependent on the composition of the rock, but at low pressures depends upon the compactness or state of aggregation as well as upon the composition.

In dealing with the elasticity of rock types, we take no account of extrusive rocks. It is to be noted, however, that such rocks if holocrystalline would have at high pressures the same elastic constants as intrusive rocks of the same composition; a rhyolite would not differ appreciably from a granite, unless it contained glass, nor, under the same conditions would an andesite differ from a diorite, nor a diabase from a gabbro. Moreover, we have little to say concerning the sedimentary or metamorphic rocks, but limit our discussion to the principal types of intrusives. This is partly because we have few experimental data for the former rocks, and also because, as a class, the igneous plutonic rocks constitute the greater part of the earth's crust.

Rigidity.—The rigidity is defined as the resistance to elastic deformation or change of shape; it is here denoted by the letter R, and like K is of the same dimensions as pressure, *e.g.*, megabars. If Poisson's ratio (the ratio of the lateral extension to the longitudinal contraction of a column subjected to a thrust) were known, the rigidity of a solid could be calculated from the bulk modulus by the well-known relation:

$$R = K\frac{3(1-2\sigma)}{2(1+\sigma)} \qquad (7)$$

σ being Poisson's ratio. For highly elastic solids, that is for solids which to a high degree regain their original size and shape when stress is removed, the value of σ is generally assumed to

be about 1/4. It is not unlikely that the plutonic rocks, which on the whole are compact, (essentially) isotropic, and highly elastic even at low pressures, may, especially at high hydrostatic pressures, fall into the class of substances for which σ is about 1/4. A still more satisfactory estimate of the value of σ for rocks can be obtained directly from seismological data. From the ratio of the velocities at which the longitudinal and the transverse earthquake waves travel through the earth (*v. infra*) it may be seen that σ is nearly constant and equal to 0.27 for all depths.[44] It is interesting to note that for wrought iron (which in its elastic properties does not differ appreciably from steel) Poisson's ratio as determined by direct experiment[45] is about 0.28, and is thus very nearly the same as the value indicated for rocks. On the other hand, it must be admitted that at comparatively low pressures many rocks may show a somewhat smaller σ. For six different granites Adams and Coker[45] find the value of Poisson's ratio to vary from 0.20 to 0.26; but these are the very rocks which tend to show the greatest abnormality as regards change of compressibility at low pressures. The more basic rocks, which often maintain their high elasticity down to low pressures, are found by Adams and Coker to have a value of σ deviating very little from 0.26, except for one—presumably not isotropic—of the two specimens of New Glasgow gabbro.

But although there may be considerable variation in Poisson's ratio at very low pressures, there seems to be no reason to doubt that this factor for all rocks at moderate or high pressures is close to 0.27. Assuming this value for σ, we have from equation (7):

$$R = 0.543\,K = \frac{0.543}{\beta} \tag{8}$$

and the values of R in Table XI have been calculated in accordance with this relation.

The rigidity of the various rocks exhibits the same general trend as the bulk modulus; which follows, of course, from the fact that if σ is constant the two quantities except at very low pressures are simply proportional to each other. Both rigidity and

[44] $\sigma = \frac{s^2 - 2}{2s^2 - 2}$, where s is the ratio of the times required for the transverse and longitudinal waves, respectively, to reach a given point. *Cf.* K. Zoeppritz and L. Geiger, *Nachr. Gesell. Wiss. Göttingen. Math.-Phys. Klasse*, 1909, 4, 428.

[45] F. D. Adams and E. G. Coker, *op. cit.*, p. 69.

bulk modulus increase steadily with increasing basicity, except for the syenite, which, although following granodiorite, has a somewhat lower rigidity.

From the values here given it is evident that the rigidity of the more important formations of the earth's crust is considerably higher than has generally been supposed. This is especially the case when we confine our attention to pressures above 1000 megabars, that is to that part of the crust lying below 2 or 3 kilometres, and neglect the uppermost thin layer, where, particularly for the granites, the elasticity may be considerably smaller. The more compact basic rocks, however, may have nearly the same compressibility and rigidity at the surface as at lower levels. The rigidity of all ordinary igneous rocks is greater than that of plate glass; that of the granites is slightly larger, while that of a peridotite or pyroxenite is about twice as much as glass. The latter rocks have nearly two-thirds the rigidity of steel (or iron).

The average composition of surface rocks is, as noted above, practically identical with the granodiorite of Table XI. If we suppose that this same average composition persists to lower levels, the rigidity of the earth's crust would be about 0.3×10^6 and its bulk modulus about 0.6×10^6. If, however, we incline to the view that the important part of the crust is largely gabbroic, the rigidity would be about 0.5×10^6.

It has long been known that the rigidity of the earth as a whole is remarkably high. At one time the rigidity was commonly stated to lie between that of glass and that of steel. Modern estimates, however, incline to the higher value. Thus Shida[46] states that the tidal deformation of the crust and the Chandler period of free nutation of the pole yield values close to 1.2×10^6 megabars, while Michelson[47] obtains 0.86×10^6, almost exactly the rigidity of steel. No silicate rock can have so high a rigidity at pressures within the range of possible experimentation. The rigidity of the more basic rocks, however, is not much below the required amount, and would doubtless be notably higher at pressures corresponding to a depth of several hundred kilometres. The high rigidity of the earth finds its explanation, of course, in the fact that the value obtained by the various methods is a sort of average rigidity for the earth as a whole, and that the rigidity of the deeper parts of the

[46] T. Shida, *Mem. Coll. Sci. and Eng.*, 1912, **4**, 116.

[47] A. A. Michelson, *Astrophys. Journ.*, 1914, **39**, 105–38.

earth, whether silicious or metallic, is made higher by the enormous pressure at these great depths.

The increase of rigidity with increasing pressure is not to be confused with the increase of strength, that is the increased resistance to flow, which as shown by the measurements of Adams [48] is very considerable. These phenomena, while probably related, are separate and distinct effects.

It remains to discuss briefly the bearing of our values for the elastic constants of rocks on the propagation of earthquakes.

VELOCITY OF EARTHQUAKE WAVES.

The velocity at which seismic waves are transmitted through the earth is intimately connected with the elastic constants of the material of which the earth is composed. As is well known, there are two kinds of waves transmitted *through* the earth, one characterized by longitudinal vibrations and travelling with a velocity:

$$U_P \sqrt{\frac{K + \frac{4}{3} R}{\rho}} \qquad (9)$$

and the other characterized by transverse vibrations travelling at a lower velocity, namely,

$$U_S \sqrt{\frac{R}{\rho}} \qquad (10)$$

Here U is the velocity in cm./sec., ρ is the density in g./cm.^{3}, and K and R are to be taken in dynes/cm.^{2}, that is 10^6 times the values given in megabars. Making use of the relation between R and β as given in equation (8), we have

$$U_P = \frac{13.13}{\sqrt{10^6 \beta \rho}} \qquad (9a)$$

$$U_S = \frac{7.37}{\sqrt{10^6 \beta \rho}} \qquad (10a)$$

in which β is now expressed in our customary unit, that is the reciprocal megabar, and U is in km./sec.

By means of equations (9a) and (10a) the velocities of the two kinds of earthquake waves for the typical igneous rocks were calculated and the results are shown in Table XI. The densities required for the calculation were computed for each rock from

[48] F. D. Adams, *J. Geol.*, 1912, **20**, 97–118. See also L. King, *ibid.*, pp. 119–138.

the mineral densities by multiplying the volume-percentage of each mineral by the corresponding density and then adding these products together.

With increasing basicity the velocity of both waves increases (there being again a slight reversal at syenite) until the ultra-basic rocks are reached, but decreases on passing down the list to iron. The total variation in velocity is less than the variation in compressibility or rigidity because the velocity is proportional to the square root of the elasticity, and also because the greater density of the basic rocks partially compensates for their higher elasticity. The velocities are least for granite and greatest for dunite, the extreme values being 5.6 and 7.4 km./sec. for the fast wave, and 3.1 and 4.1 km./sec. for the slow one. The velocities of earthquake waves are known to vary with the distance from the source and hence with the depth to which the waves penetrate, and may be calculated from seismological data. The initial velocities, that is the velocities at or near the surface, are generally taken to be about 4 km./sec. for the transverse wave and a little above 7 km./sec. for the longitudinal one. These are the values corresponding to the pyroxenites or to the periodites, and are considerably above the velocities in the granitic rocks, although the velocity in granite as shown by Table XI is notably higher than had previously been supposed possible in rocks of that character. The results here given show the possibility of ascribing the high initial velocities of earthquakes to a preponderance of ultra-basic material at a comparatively small depth.

We shall not attempt at this time to discuss further the bearing of our results for the elastic constants of rocks on the question of the constitution of the earth. This subject will be taken up in a subsequent paper dealing with the earth's interior.

In conclusion we take this opportunity of acknowledging our indebtedness to Dr. Olaf Andersen, to Professor G. P. Merrill, to Mr. E. V. Shannon, and to Mr. Hans Brassler for furnishing us various mineral specimens; to Professor Frank D. Adams, who gave us specimens of several of the rocks with which he has worked; to Dr. H. E. Merwin and to Dr. N. L. Bowen for microscopical examination of many of the rocks and minerals; and to Dr. H. S. Washington for making seven complete chemical analyses for us, for the examination of many of the rocks and minerals, and for his kindly criticism and helpful suggestion.

SUMMARY.

The compressibilities of a number of minerals and igneous rocks have been determined under hydrostatic pressures up to 12,000 megabars, corresponding to a depth of 40 kilometres below the surface of the earth. The method used has already been shown to yield consistent results for metals and other solids. According to this method the specimen is completely surrounded by a liquid, kerosene, and is subjected to pressure in a thick-walled steel bomb.

The compressibility of the minerals usually falls off slightly as the pressure is increased. For the less compressible minerals, however, the change in compressibility is so small as to escape detection by the present method, the precision of which corresponds to about 0.01×10^{-6} per megabar, that is to 1 per cent. of the total compressibility of the less compressible minerals. The absolute accuracy, of course, is not so high.

In connection with the compressibility of rocks, a complication is introduced by reason of their porosity, which even in igneous rocks is in many cases enough to affect the compressibility. In order to determine the effect of porosity at pressures within the range of experiment—1000 to 12,000 megabars—the porous rocks were covered with a thin jacket of pure tin, which served to prevent the liquid from entering the pores and thus allowed the closing-in of the pores to contribute to the decrease in volume of the sample as a whole. It was found that at pressures above 2000 megabars porosity has very little effect on the compressibility; but a comparison with the results of F. D. Adams and E. G. Coker shows that certain rocks, notably the more porous ones, may have at low pressures an abnormally high compressibility.

Our results show that except for very low pressures the compressibility of a rock may be calculated directly from the known compressibility of the constituent minerals, provided that the rock is holocrystalline. In general, the compressibility increases with increasing basicity, that is with decreasing silica content. Quartz and metallic iron may be considered to stand at the two extremes as regards the compressibility of igneous rocks. In any given class of rocks the variation in compressibility at high pressures is small and almost wholly due to the variation in composition; at low pressures the variation from one rock to another in the same class is much larger and depends mainly on the degree of compactness of

the rock. The granites as a rule show at low pressures a compressibility much above what may be called the normal compressibility, but basic rocks may preserve their relatively small normal compressibility down to low pressures. At 10,000 megabars the compressibility of a typical granite is about 1.9×10^{-6} per megabar, and that of a typical gabbro is about 1.2×10^{-6}.

From the ratio of the velocities of the two kinds of waves transmitted through the earth and from the known elastic behavior of compact rocks, it is believed that the value of Poisson's ratio for rocks at moderate or high pressures is very close to 0.27. From this value and from the measured compressibility the rigidity of a number of typical rocks is calculated. The rigidity of a typical granite at 10,000 megabars is 0.3×10^{6} megabars; that of a typical gabbro is 0.5×10^{6}; and that of an ultra-basic rock, *e.g.*, a dunite, is 0.6×10^{6}. The rigidity of the earth as a whole, which is probably near that of steel (0.9×10^{6}), is higher than the value for the most rigid silicate rock, but doubtless the rigidity of a basic or ultra-basic rock is raised enough by a pressure corresponding to a few hundred miles of rock to yield the required value.

From the values of the bulk modulus (the reciprocal of the compressibility) and of the rigidity the velocities of the two kinds of waves transmitted through the earth are calculated for the various types of rocks. With increasing basicity of the rocks the velocity of the longitudinal vibrations increases steadily from the value for granite, 5.6 km./sec., to that for dunite, 7.4 km./sec., but is lower, 6.0 km./sec., for metallic iron. The initial velocity of the longitudinal vibrations is usually taken to be a little above 7 km./sec. This corresponds to a gabbro or to a pyroxenite, and furnishes an indication of basic or ultra-basic material at a relatively small depth below the surface of the earth.

The Mechanics of Newton Versus the Mechanics of Einstein. J. LEROUX. (*Comptes Rendus*, Dec. 26, 1922.)—"Most of the discussions of the principle of Einstein are vitiated by a fundamental error which should be dissipated, *viz.*, that the mechanical system of Newton is an approximation to that of Einstein. The two theories rest on entirely different principles. They give results almost alike in the case of a single moving body, but no one has the right to draw the conclusion that their consequences are always close together for systems of bodies. An affirmation of this kind is as false as a statement claiming that two curves near to each other in a limited region are constantly near together throughout their entire extent."

A Critical Review of Trouton's Law, and its Applicability to the Triple Point. SASI BHUSAN MALI. (*Phil. Mag.*, Jan., 1923.)—"The generalized Trouton law may be expressed by saying that the latent heat of change of state of a substance is proportional to the temperature of change of state, and the ratio of the two is approximately the same for all substances, provided the same kind of change of state is involved." Nernst found that this did not hold generally for vaporization. The author tries to find some circumstances under which the law is true. It certainly does not hold in the case of water for which there is no proportionality between the two quantities, temperature and latent heat, unless 40 and 16 are to be regarded as alike. When the ratio is examined for corresponding states of substances no theoretical basis of the law can be found. The author tries the triple point, but again here a wide divergence of ratios comes to light, so that he is forced to conclude, "It thus seems fairly well established that there is no particular point of temperature at which the Trouton law holds. Neither does the law seem to have any theoretical significance." G. F. S.

Relative Visibility of Spectra When an Electric Discharge is Passed through the Vapors of Alkali Amalgams. F. H. NEWMAN. (*Phil. Mag.*, Jan., 1923.)—Amalgams of sodium, potassium, lithium, rubidium and cæsium, all were used. "At 100° C., the mercury lines predominate, but at 200° C. the spectra of the alkali metals are brighter than the mercury spectrum. The sodium D-lines mask the mercury lines at the higher temperature. The potassium spectrum is very faint under all conditions. The employment of the amalgams of the alkali metals, instead of the latter, forms a useful and convenient method for obtaining the spectra of the alkali metals. A brilliant D-line radiation is obtained with sodium amalgam." Two factors are concerned in determining the relative intensities of the spectra of the two elements present, their respective ionization potentials and their vapor-pressures. For instance, the ionizing potential of mercury is higher than that of sodium and yet it is the mercury spectrum that predominates at the lower temperature.

"At low temperatures, although the vapor-pressure of mercury is large compared with that of sodium, the actual values in both cases are small. As a result, the sodium atoms in the discharge-tube will be comparatively few. The mercury atoms also will probably be so far apart that the electrons attain the energy necessary to ionize the mercury atoms before they collide inelastically with any of the atoms. At low vapor-pressures, therefore, it is probable that the same proportion of mercury and sodium atoms present in the tube will be ionized. The intensity of the radiation emitted must be intimately connected with the concentration of the radiating atoms in the source, so that as the concentration of the radiating mercury atoms is greater than that of the sodium, we should expect the mercury lines to predominate." G. F. S.

A SURVEY OF EXISTING CRYSTAL STRUCTURE DATA.*†

BY

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DOUBLE HALIDES AND CYANIDES.

DOUBLE CYANIDES OF DI- AND UNIVALENT ELEMENTS.—The crystal structures have been studied [122] of cubic cyanides of the type $K_2R(CN)_4$, where R is either zinc, cadmium or mercury. The coördinate positions of the atoms within the unit cube which contains eight chemical molecules (Fig. 18) have been established from spectrographic and Laue photographic data to be:

Potassium: $\frac{1}{8}\frac{3}{8}\frac{3}{8}$; $\frac{7}{8}\frac{1}{8}\frac{3}{8}$; $\frac{3}{8}\frac{5}{8}\frac{1}{8}$; $\frac{3}{8}\frac{3}{8}\frac{5}{8}$; $\frac{1}{8}\frac{5}{8}\frac{5}{8}$; $\frac{1}{8}\frac{1}{8}\frac{8}{8}$; $\frac{5}{8}\frac{1}{8}\frac{1}{8}$; $\frac{5}{8}\frac{5}{8}\frac{5}{8}$; $\frac{3}{8}\frac{3}{8}\frac{1}{8}$; $\frac{1}{8}\frac{7}{8}\frac{8}{8}$; $\frac{5}{8}\frac{1}{8}\frac{7}{8}$; $\frac{3}{8}\frac{3}{8}\frac{9}{8}$; $\frac{7}{8}\frac{5}{8}\frac{5}{8}$; $\frac{7}{8}\frac{7}{8}\frac{8}{8}$; $\frac{5}{8}\frac{7}{8}\frac{7}{8}$; $\frac{9}{8}\frac{9}{8}\frac{9}{8}$.

Divalent metal atom: 000; $\frac{1}{2}\frac{1}{2}0$; $0\frac{1}{2}\frac{1}{2}$; $\frac{1}{2}0\frac{1}{2}$; $\frac{1}{4}\frac{1}{4}\frac{1}{4}$; $\frac{3}{4}\frac{3}{4}\frac{1}{4}$; $\frac{3}{4}\frac{1}{4}\frac{3}{4}$; $\frac{1}{4}\frac{3}{4}\frac{3}{4}$.

Carbon: uuu; $u+\frac{1}{2},u+\frac{1}{2},u$; $u+\frac{1}{2},u,u+\frac{1}{2}$; $u,u+\frac{1}{2},u+\frac{1}{2}$; $u\bar{u}\bar{u}$; $u+\frac{1}{2},\frac{1}{2}-u,\bar{u}$; $u+\frac{1}{2},\bar{u},\frac{1}{2}-u$; $u,\frac{1}{2}-u,\frac{1}{2}-u$; $\bar{u}u\bar{u}$; $\frac{1}{2}-u,u+\frac{1}{2},\bar{u}$; $\frac{1}{2}-u,u,\frac{1}{2}-u$; $\bar{u},u+\frac{1}{2},\frac{1}{2}-u$; $\bar{u}\bar{u}u$; $\frac{1}{2}-u,\frac{1}{2}-u,u$; $\frac{1}{2}-u,\bar{u},u+\frac{1}{2}$; $\bar{u},\frac{1}{2}-u,u+\frac{1}{2}$; $\frac{1}{4}-u,\frac{1}{4}-u,\frac{1}{4}-u$; $\frac{3}{4}-u,\frac{3}{4}-u,\frac{1}{4}-u$; $\frac{3}{4}-u,\frac{1}{4}-u,\frac{3}{4}-u$; $\frac{1}{4}-u,\frac{3}{4}-u,\frac{3}{4}-u$; $\frac{1}{4}-u,u+\frac{1}{4},u+\frac{1}{4}$; $\frac{3}{4}-u,u+\frac{3}{4},u+\frac{1}{4}$; $\frac{3}{4}-u,u+\frac{1}{4},u+\frac{3}{4}$; $\frac{1}{4}-u,u+\frac{3}{4},u+\frac{3}{4}$; $u+\frac{1}{4},\frac{1}{4}-u,u+\frac{1}{4}$; $u+\frac{3}{4},\frac{3}{4}-u,u+\frac{1}{4}$; $u+\frac{3}{4},\frac{1}{4}-u,u+\frac{3}{4}$; $u+\frac{1}{4},\frac{3}{4}-u,u+\frac{3}{4}$; $u+\frac{1}{4},u+\frac{1}{4},\frac{1}{4}-u$; $u+\frac{3}{4},u+\frac{3}{4},\frac{1}{4}-u$; $u+\frac{3}{4},u+\frac{1}{4},\frac{3}{4}-u$; $u+\frac{1}{4},u+\frac{3}{4},\frac{3}{4}-u$.

Nitrogen: a similar set of positions in v.

For the zinc salt the position of the centre of the cyanide group lies close to $\frac{1}{2}$ $(u+v)=0.37$. The data from the other two salts are in harmony with this result, but are of a much scantier nature. With these two variable parameters and the present lack of knowledge concerning scattering, it was not possible to determine with certainty the exact positions of the nitrogen and carbon atoms. For the double zinc cyanide, however, the data accord best with one atom at about 0.34 and the other at 0.40.

Potassium Zinc Cyanide, $K_2Zn(CN)_4$.—The length [122] of the side of the unit cube is found to be: $a=12.54$A.U.

Potassium Cadmium Cyanide, $K_2Cd(CN)_4$.—The length [122] of the side of the unit cube is: $a=12.84$A.U.

* Communicated by Dr. Arthur L. Day, Director of the Geophysical Laboratory and Associate Editor of this JOURNAL.

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Potassium Mercury Cyanide, $K_2Hg(CN)_4$.—The length [122] of the side of the unit cube: $a = 12.76$A.U.

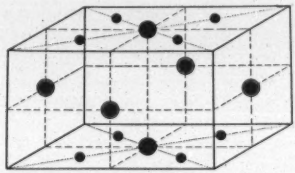
DOUBLE HALIDES OF DI- AND UNIVALENT ELEMENTS.—Studies have been made of tetragonal crystals of the type R_2XCl_4, where R is ammonium or potassium and X is platinum or palladium. The following arrangement (Fig. 19) was found to be the simplest which would account for spectrum and Laue photographic data.[101]

Potassium or Nitrogen:	$0\frac{1}{2}\frac{1}{2}$; $\frac{1}{2}0\frac{1}{2}$.
Platinum or Palladium:	000.
Chlorine:	$uu0$; $\bar{u}u0$; $u\bar{u}0$; $\bar{u}\bar{u}0$.

For each crystal examined u has been found to be close to 0.23.

Potassium Chloroplatinite, K_2PtCl_4.—The dimensions [101] of

FIG. 19.



The arrangement of the atoms within the unit tetragonal prism assigned to potassium chloroplatinite. Platinum atoms are shown by large, chlorine by small black circles.

the unit prism described above are found to be: $a = 6.99$A.U., $c = 4.13$A.U.

Potassium Chloropalladite, K_2PdCl_4.—The dimensions [101] of the unit prism are: $a = 7.04$A.U., $c = 4.10$A.U.

Ammonium Chloropalladite, $(NH_4)_2PdCl_4$.—The dimensions [101] of the unit prism are: $a = 7.21$A.U., $c = 4.26$A.U.

DOUBLE HALIDES OF QUADRI- AND UNIVALENT ELEMENTS.—The structures of the cubic crystals ammonium chloroplatinate, potassium and ammonium chlorostannates and ammonium fluosilicate have been studied. It is conclusively shown [123, 124] that these isomorphous compounds have the following arrangement of their atoms within the unit cube (Fig. 16).

Quadrivalent atoms: 000; $\frac{1}{2}\frac{1}{2}0$; $0\frac{1}{2}\frac{1}{2}$; $\frac{1}{2}0\frac{1}{2}$.

Univalent metal atoms (or nitrogen atom of an ammonium group): $\frac{1}{4}\frac{1}{4}\frac{1}{4}$; $\frac{3}{4}\frac{3}{4}\frac{1}{4}$; $\frac{3}{4}\frac{1}{4}\frac{3}{4}$; $\frac{1}{4}\frac{3}{4}\frac{3}{4}$; $\frac{3}{4}\frac{3}{4}\frac{3}{4}$; $\frac{3}{4}\frac{1}{4}\frac{1}{4}$; $\frac{1}{4}\frac{3}{4}\frac{1}{4}$; $\frac{1}{4}\frac{1}{4}\frac{3}{4}$.

Halogen atoms: The same twenty-four equivalent positions that have defined the positions of the nitrogen atoms in hexammonates of the nickel halides (see March issue, page 363).

Hydrogen atoms: The thirty-two equivalent hydrogen atoms within the unit cell of the ammonium compounds will presumably have the arrangement that has been designated[122] as 32a. Their positions cannot, however, be determined from the diffraction data. It has also been reported[119] that potassium chloroplatinate has this same structure.

Potassium Chloropalatinate, K_2PtCl_6.—No data have yet been published.[119] Apparently upon the basis of powder measurements u has been given a value around 0.16. This parameter is the same in the ammonium and potassium chlorostannates,[124] and it is probable that the ammonium and potassium chloroplatinates are similar. Inasmuch as a complete study[123] of the structure of the ammonium salt has shown that $u = \pm 0.24$, u for the potassium chloroplatinate may be expected to be near to 0.24. A further investigation is consequently to be desired.

Ammonium Chloroplatinate, $(NH_4)_2PtCl_6$.—The value[123] of u for chlorine lies between 0.22 and 0.24. The length of the side of the unit cube is given as: $a = 9.84$A.U.

Potassium Chlorostannate, K_2SnCl_6.—The chlorine parameter[124] must be less than 0.25 and lie close to 0.245. The length of the side of the unit cube is given as: $a = 9.99$A.U.

Ammonium Chlorostannate, $(NH_4)_2SnCl_6$.—The parameter[124] u is the same (around 0.245) as for the potassium salt. The length of the side of the unit cube is likewise almost the same: $a = 10.05$A.U.

Ammonium Fluosilicate, $(NH_4)_2SiF_6$.—This crystal is dimorphous, one form being cubic, the other hexagonal. Only the former has been investigated.[125] It is concluded that u is close to 0.205. The length of the side of the unit cube is determined to be: $a = 8.38$A.U.

SUMMARY OF THE STRUCTURES OF SIMPLE AND COMPLEX HALIDES, CYANIDES, ETC.—The structures of the crystals of this group that have been studied are summarized in Table IV.

NITRATES, CHLORATES, BROMATES.

Sodium Nitrate, $NaNO_3$.—Neither of the two investigations[86, 126] of the structure of sodium nitrate makes adequate use of the results of the theory of space-groups. It is quite certain nevertheless that the correct atomic arrangement has been

TABLE IV.

The Crystal Structures of Simple and Complex Halides, Cyanides, Etc.

Compound.	System of Symmetry.	Type of Arrangement.	Text Figure of Unit Cell.	Dimensions of Unit Cell.	Class of Determination.
LiH	Cubic	NaCl	Fig. 6	$a = 4.10$	IIA
LiF	Cubic	NaCl	Fig. 6	$a = 4.14, 4.02$	IIA
LiCl	Cubic	NaCl	Fig. 6	$a = 5.17$	IA
LiBr	Cubic	NaCl	Fig. 6	$a = 5.48$	IA
LiI	Cubic	NaCl	Fig. 6	$a = 6.06$	IA
NaF	Cubic	NaCl	Fig. 6	$a = 4.61, 4.68$	IA
NaCl	Cubic	NaCl	Fig. 6	$a = 5.628$	IIA
NaBr	Cubic	NaCl	Fig. 6	$a = 5.95, 6.02$	IA
NaI	Cubic	NaCl	Fig. 6	$a = 6.47, 6.50$	IA
KF	Cubic	NaCl	Fig. 6	$a = 5.36, 5.38$	IA
KCl	Cubic	NaCl	Fig. 6	$a = 6.26$	IIA
KBr	Cubic	NaCl	Fig. 6	$a = 6.59, 6.60$	IA
KI	Cubic	NaCl	Fig. 6	$a = 7.11, 7.10$	IA
RbF	.....	.....	.....	.....	.....
RbCl	Cubic	NaCl	Fig. 6	$a = 6.60$	IA
RbBr	Cubic	NaCl	Fig. 6	$a = 6.93$	IIA
RbI	Cubic	NaCl	Fig. 6	$a = 7.36, 7.31$	IA
CsF	Cubic	NaCl	Fig. 6	$a = 6.03$	IA
CsCl	Cubic	CsCl	Fig. 11	$a = 4.12$	IIA
CsBr	Cubic	CsCl	Fig. 11	$a = 4.30$	IA
CsI	Cubic	CsCl	Fig. 11	$a = 4.55$	IA
TlCl	Cubic	CsCl	Fig. 11	$a = 3.85$	IIA
CuCl	Cubic	ZnS	Fig. 9	$a = 5.49, 5.36$	IA
CuBr	Cubic	ZnS	Fig. 9	$a = 5.82, 5.75$	IA
CuI	Cubic	ZnS	Fig. 9	$a = 6.10, 6.07$	IA
AgCl	Cubic	NaCl	Fig. 6	$a = 5.56, 5.52$	IIA
AgBr	Cubic	NaCl	Fig. 6	$a = 5.78$	IIA
AgI (low)	Hexagonal	ZnO	Fig. 7	$a = 4.59$; $c = 7.53$	IA

AgI (high)	Cubic	Great complexity			IA
NH_4Cl (low)	Cubic	NH_4Cl	Fig. 12	$a = 3.859$	IA
NH_4Cl (high)	Cubic	NH_4Cl type		$a = 6.53$ at 250°	Incomplete
NH_4Br (low)	Cubic	NH_4Cl type	Fig. 12	$a = 3.988$	IIA
NH_4Br (high)	Cubic	NaCl type		$a = 6.90$ at 250°	Incomplete
NH_4I	Cubic	NaCl type			Incomplete
PH_4I	Tetragonal	PH_4I	Fig. 13	$a = 6.34$; $c = 4.62$	IP
$N(CH_3)_4I$	Tetragonal				IB
KCN	Cubic	NaCl type		$a = 6.54$, 6.55	IA
CaF_2	Cubic	CaF_2	Fig. 14	$a = 5.452$, 5.478, 5.455, 5.49	IIA
BaF_2	Cubic	CaF_2	Fig. 14	$a = 6.83$	IIA
CdI_2	Hexagonal	$Mn(OH)_2$	Fig. 8	$a = 4.24$; $c = 6.84$	IP
$NaHCl_2$	Cubic	Pyrite type	Fig. 15	$a = 7.89$	IA
$NiCl_2.6NH_3$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 10.09$	IA
$NiBr_2.6NH_3$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 10.48$	IA
$NiI_2.6NH_3$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 11.01$	IA
$CsCl_3I$	Hexagonal	$CsCl_3I$	Fig. 17	$a = 5.46$; $a = 70° 42'$	IB
K_2I_3	Monoclinic				IIB
$K_2Zn(CN)_4$	Cubic	$K_2Zn(CN)_4$	Fig. 18	$a = 12.54$	IA
$K_2Cd(CN)_4$	Cubic	$K_2Zn(CN)_4$	Fig. 18	$a = 12.84$	IA
$K_2Hg(CN)_4$	Cubic	$K_2Zn(CN)_4$	Fig. 18	$a = 12.76$	IA
K_2PtCl_4	Tetragonal	K_2PtCl_4	Fig. 19	$a = 6.99$; $c = 4.13$	IP
K_2PdCl_4	Tetragonal	K_2PtCl_4	Fig. 19	$a = 7.04$; $c = 4.10$	IP
$(NH_4)_2PdCl_4$	Tetragonal	K_2PtCl_4	Fig. 19	$a = 7.21$; $c = 4.26$	IP
$(NH_4)_2PtCl_4$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 9.843$	IA
K_2SnCl_6	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 9.99$	IA
$(NH_4)_2SnCl_6$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 10.05$	IA
$(NH_4)_2SiF_6$	Cubic	$NiCl_2.6NH_3$	Fig. 16	$a = 8.38$	IA

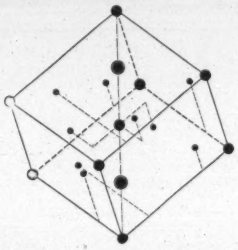
given not only because of the excellent accord with the experimental data, but also on account of the agreement with the more completely studied members of the calcite group. A single spectrometric observation [86] upon a cleavage face has been said to agree with a structure identical with that previously assigned to calcite. A study [126] of spectrum and Laue photographs gave results in substantial agreement.

The unit cell is a rhombohedron containing two chemical molecules in the following coördinate positions:

Oxygen: 000; $\frac{1}{2}\frac{1}{2}\frac{1}{2}$.
Sodium: $\frac{1}{4}\frac{1}{4}\frac{1}{4}$; $\frac{3}{4}\frac{3}{4}\frac{3}{4}$.
Nitrogen: $u\bar{u}0$; $\bar{u}0u$; $0u\bar{u}$; $\frac{1}{2}-u,u+\frac{1}{2},\frac{1}{2}$; $u+\frac{1}{2},\frac{1}{2},\frac{1}{2}-u$; $\frac{1}{2},\frac{1}{2}-u,u+\frac{1}{2}$.

The spectrometric measurement is held to agree with a value of

FIG. 20.



The unit rhombohedron of the "calcite arrangement." No attempt has been made to indicate the correct shape of this unit. The carbon atoms (of calcite) are shown by large, the oxygen atoms by small black circles.

u that is close to $\frac{1}{4}$. The study of Laue photographs also shows that u must lie very near indeed to 0.25. The length of the side of this unit rhombohedron (Fig. 20), the angle between the axes of which is 48°6′, is found to be 6.320A.U. A somewhat better idea of the marshalling of the atoms within the crystal is furnished by the representation of the larger construction cell of atoms which accompanies the spectrometric studies.

SODIUM CHLORATE AND SODIUM BROMATE.—In one of the two investigations carried out upon these cubic crystals powder photographic data [127] were used, in the other spectrometric measurements.[128] Both agree in deducing the same atomic arrangement but they differ in the positions assigned to the oxygen atoms. This is not surprising, however, when it is realized that the five

variable parameters that exist can only be established by a procedure of trial. The coördinates of the positions of the atoms within the unit cube which contains four chemical molecules must be as follows:

Sodium: uuu, $u+\frac{1}{2},\frac{1}{2}-u,\bar{u}$; $\bar{u},u+\frac{1}{2},\frac{1}{2}-u$; $\frac{1}{2}-u,\bar{u},u+\frac{1}{2}$.
Halogen: a similar set of positions in v.

Oxygen: the twelve generally equivalent positions of the space-group T^4: xyz; $x+\frac{1}{2},\frac{1}{2}-y,\bar{z}$; $\bar{x},y+\frac{1}{2},\frac{1}{2}-z$; $\frac{1}{2}-\bar{x},y,z+\frac{1}{2}$;
zxy; $\bar{z},x+\frac{1}{2},\frac{1}{2}-y$; $\frac{1}{2}-z,\bar{x},y+\frac{1}{2}$; $z+\frac{1}{2},\frac{1}{2}-x,\bar{y}$;
yzx; $\frac{1}{2}-y,\bar{z},x+\frac{1}{2}$; $y+\frac{1}{2},\frac{1}{2}-z,\bar{x}$; $\bar{y},z+\frac{1}{2},\frac{1}{2}-x$.

Sodium Chlorate, $NaClO_3$.—The length of the side of the unit cube has been taken as: $a=6.55$A.U.[127] and $a=6.56$A.U.[128] The latter of these two values is calculated from the density $\rho=2.49$. In the two investigations the values of the parameters defining the positions of the atoms have been chosen as: $u=0.083$ or 0.08, $v=0.417$ or 0.43, $x=0.50$ or 0.30, $y=0.417$ or 0.59, $z=0.306$ or 0.47. It will be observed that there is close agreement in the positions of the sodium and chlorine atoms, but that the oxygen atoms are differently placed.

Sodium Bromide, $NaBrO_3$.—The length of the side of the unit cube is given as: $a=6.74$A.U.[127] and $a=6.71$A.U.[128] The latter is calculated from a density $\rho=3.30$. In one of the investigations [127] the same coördinate positions are considered to apply to both the bromate and the chlorate; for the other $u=0.09$, $v=0.41$, $x=0.30$, $y=0.60$ and $z=0.47$.

There can be little doubt of the general correctness of this structure and it is also likely that the sodium and halogen atoms have been placed near to their true positions. Of the two assignments of position to the oxygen atoms, those based upon spectrometric measurements are in somewhat better agreement with the experimental data, though it must be remembered that their precise values will be influenced by the scattering assumptions used. It is interesting to note that for those reflections which give separate lines in the powder photographs both sets of atomic positions are in about equally good agreement with the experimental data: The less satisfactory character of the first assignment appears in those planes which can only be separately studied with spectrometric or Laue photographic methods.

THE NITRATES OF BARIUM, STRONTIUM, LEAD AND CALCIUM.—From an investigation [129] which has not been published in detail, it is said that spectrographic measurements upon the first three of these crystals show four chemical molecules to be associated with the unit cube. From this fact and Laue photographic data they are assigned to the space-group T_h^6 (the same as that of pyrite); hence the atoms have the following coördinate positions:

Divalent metal atoms: 000; $\frac{1}{2}\frac{1}{2}0$; $0\frac{1}{2}\frac{1}{2}$; $\frac{1}{2}0\frac{1}{2}$.

Nitrogen atoms: uuu; $u+\frac{1}{2},\frac{1}{2}-u,\bar{u}$; $\bar{u},u+\frac{1}{2},\frac{1}{2}-u$; $\frac{1}{2}-u,\bar{u},u+\frac{1}{2}$; $\bar{u}\bar{u}\bar{u}$; $\frac{1}{2}-u,u+\frac{1}{2},u$; $u,\frac{1}{2}-u,u+\frac{1}{2}$; $u+\frac{1}{2},u,\frac{1}{2}-u$.

Oxygen atoms: the twenty-four generally equivalent positions of the [132] space-group T_h^6.

The values of these four parameters were not determined and it would scarcely seem feasible to try to do so at the present time, but it is said that $u=\frac{1}{4}$, and $x=\frac{1}{4}$, $y=\frac{1}{4}$, $z=0$, may be chosen as approximate positions. The marked hemihedral character of the Laue photographs must preclude the possibility of these coördinate values being exactly fulfilled. No determination of dimensions accompany this paper. In spite of the fragmentary character of the reported results, it is probably correct in its general assignment of structure. Recently powder photographic data [130] have been collected from these crystals and from calcium nitrate. A structure is given of the same type as the one just described and an assignment of parameters for the nitrogen and oxygen atoms made. These crystals are even less favorable for such an estimation of variable parameters than was sodium chlorate, because all of the atoms whose positions are to be determined have relatively slight scattering powers. In view of the difficulties apparently encountered by powder photographic data in placing the oxygen atoms in sodium chlorate, the nitrogen and oxygen positions assigned to these crystals must be of uncertain value. The lengths of the sides of the unit cubes for these crystals are determined to be:

Crystal.	$a \times 10^{-8}$ cm.
$Ca(NO_3)_2$	7.60
$Sr(NO_3)_2$	7.81
$Ba(NO_3)_2$	8.11
$Pb(NO_3)_2$	7.84

HYDRATES AND AMMONATES OF NITRATES, CHLORATES, ETC.—Two of these crystals have been studied: Zinc bromate hexahydrate and nickel nitrate hexammonate. Both have been investigated with spectrographic and Laue photographic data. The arrangement of their atoms can be determined with entire definiteness though it is not now practicable to try to locate some of these atoms with precision. The coördinates defining the atomic positions are the same for both (except for the different numbers of hydrogen atoms in the two compounds) and the same, as far as they go, as the coördinates of the atoms of barium, strontium and lead nitrates which have just been considered. Consequently the coördinates will not be repeated. The oxygen atoms of the water groups and the nitrogen atoms of the ammonia groups will also occupy the generally equivalent positions of T_h^6 with values of x, y and z which are different from those of the oxygen atoms of the anion groups. The hydrogen atoms must likewise occupy two or three sets, as the case may be, of generally equivalent positions with still different coördinate values.

Zinc Bromate Hexahydrate,[131] $Zn(BrO_3)_2 \cdot 6H_2O$.—The length of the side of the unit cube containing four chemical molecules was stated to be: $a = 10.31$A.U. No estimation is offered of the variable parameters within this structure.

Nickel Nitrate Hexammonate,[132] $Ni(NO_3)_2 \cdot 6NH_3$.—The length of the side of the unit cube was found to be: $a = 10.96$A.U. From the general character of the Laue photographic data it is probable that the coördinates for the ammonia nitrate atoms are close to vvv, where v is about $\frac{1}{4}$. Other than this no estimate of the variable parameters within the structure can be offered. This compound is the same as the one which has previously been described as $Ni(NO_3)_2 \cdot 4NH_3 \cdot 2H_2O$.

CARBONATES.

Measurements have been recorded upon the related minerals: Calcite, dolomite, rhodochrosite and siderite. Calcite, the best studied, has been subjected to a detailed treatment by various investigators.[86, 133, 134, 135] They all agree in giving to it, and to the other members of the group, the same atomic arrangement which has been discussed for sodium nitrate with carbon atoms replacing the nitrogen (Fig. 20).

Calcite, $CaCO_3$.—The original determination[86] of structure was made with spectrometric measurements. This has since been

established with apparent certainty and positions for the oxygen atoms deduced with Laue photographic data.[134,135] There is, however, a sharp difference of opinion concerning the value of the parameter u for oxygen. The original spectrometric measurements[86] and one of the studies based upon Laue photographs[134] agree with one another in making u about 0.30. The later, and more accurate, spectrometric observations,[133] and the other work based upon Laue photographs,[135] find that u must lie close to 0.25.

This discrepancy calls for some discussion. In the first place, the spectrometric results, whatever may be their value for this purpose, are definitely in favor of the lower parameter. In one of the determinations[134] of the positions of the oxygen atoms, based upon Laue photographic data, a more or less quantitative use seems to have been made of the so-called laws of scattering. In view of the probably only qualitative correctness of these laws it would seem that this procedure does not necessarily form a faithful guide for placing atoms. In the other determination,[135] an attempt was made to avoid the use of these "laws" where possible and in no case to make anything but a roughly qualitative use of them. Thus only planes to which no atoms but oxygen contributed were used for locating these atoms and the only intensity criteria that were called into play were observations of the existence of large differences of intensity between pairs of reflections. It was only after a prolonged study of Laue photographs that data were found to decide between the two positions of 0.25 and 0.30; eventually reflection data were met that appear in flat contradiction with the higher value of u. This latter determination[133, 135] of $u = 0.25$, in its avoidance of a quantitative use of scattering assumptions known to hold only qualitatively, thus is the safer one.

The spacing between cleavage planes is generally taken as the X-ray standard of wave-length. The length of the correct unit rhombohedron containing two chemical molecules (Fig. 20) is 6.360A.U.; the angle between the axes 46°6′. In one of the papers on calcite and the other members of this group,[135] the dimensions of the unit cell were incorrectly stated. As in the case of sodium nitrate, a better idea of the atomic arrangement can be obtained from viewing a larger construction rhombohedron.[86] This is the crystallographically chosen unit, the equal angles between rhombohedral axes being 101°55′.

Rhodochrosite, $MnCO_3$.—Spectrometer,[86] spectrographic and Laue photographic data[135] have been studied. The structure is the same as that of calcite except that a different value is to be assigned to the oxygen parameter. The Laue photographs have been used, as in one of the treatments[135] of calcite, to place the oxygen atoms with only a qualitative use of one of the "laws" of scattering. The value thus chosen * is around 0.27. The length of the side of the unit rhombohedron containing two chemical molecules is 5.836A.U.; the angle between the axes 47°45′.

Siderite, $FeCO_3$.[86, 135]—The Laue photographic data,[135] though not so good as those for the two previous salts, showed no difference between the parameter for this crystal and for rhodochrosite. It may consequently be taken as probably close to 0.27. The length of the side of the unit rhombohedron (Fig. 20) also approaches that for rhodochrosite: $a = 5.822$A.U. The angle between the axes is 47°45′.

Dolomite, $CaMg(CO_3)_2$.—The single reflection spectrum[86] that has been taken is said to agree with a structure similar to that of the other calcite group minerals.

SILICATES.

Attempts have been made to study the atomic arrangement in two silicates, olivine and garnet. Both have made some use of the results of the theory of space-groups, though in their present forms neither is entirely satisfactory.

Olivine,[136] $(Mg,Fe)_2SiO_4$.—The ratio of magnesium to iron in the specimen studied was about six to one. Many Laue photographs in different orientations were prepared from this orthorhombic crystal and the planes corresponding to the observed spots identified. Use of these data does not seem to have been made in the subsequent attempt to establish the space-group of olivine. Reflection spectra give a unit cell with dimensions corresponding to the accepted axial ratio. They are also thought to show that the corresponding space-group is probably V_h^{16}. In view of the apparent failure to establish this cell as the simplest one, and not merely one of a number of crystallographically possible units, there is some doubt concerning the uniqueness of this determination.

* The early estimation[86] of u, based upon approximate spectrometric measurements, placed the oxygen parameter for the other crystals than calcite considerably above 0.30.

Garnet.[137]—Crystals of garnet have the general composition $R_3''R_2'''(SiO_4)_3$, where R'' and R''' can be various di- and trivalent metallic elements. The one measured was a ferrous aluminum garnet in which about one-third of the iron was replaced by manganese. Both spectrographic and Laue photographic observations were made upon this crystal though. data have never been published. There appears to be some conflict between the structure and the determination of the number of molecules within the unit cell. A study in the light of recent procedure probably is required before the results of this investigation can be taken as firmly established.

SULFATES, MOLYBDATES, TUNGSTATES, ETC.

Alkali Sulfates.[138]—Spectrometric observations have been made upon crystals of the isomorphous series of the sulfates of potassium, ammonium, rubidium and cæsium, primarily for the purpose of testing the hypothesis of valency-volume. These measurements will form a valuable source of data in the future, but at present they are quite incapable by themselves of establishing the arrangement of the atoms in these crystals.

Silver Molybdate, Ag_2MoO_4.—The structure[139] of crystals of silver molybdate has been established from spectrographic and Laue photographic data. The coördinate positions of the atoms within the unit cube (Fig. 18) are the same as those of the corresponding atoms of $K_2Zn(CN)_4$: Silver and potassium atoms, molybdenum and zinc atoms, and oxygen and either the carbon or nitrogen atoms having the same coördinates. The parameter for oxygen could not be accurately determined, probably because of the much greater scattering powers of the other atoms, but it is shown that u must have a value that is close to 0.37 and can scarcely be greater than 0.40 or less than 0.34. The length of the side of this unit cube which contains eight molecules is found to be: $a = 9.26$A.U.

Scheelite, $CaWO_4$.—Spectrometric measurements[140] from various faces have been made upon crystals of scheelite. Such observations are not sufficient to determine the structure of this tetragonal crystal with any degree of certainty.

Wulfenite, $PbMoO_4$.—The same treatment[140] has been given to this crystallographically related crystal.

Alums.—An early attempt[141] was made to obtain the structure of potash alum. Spectrometer data[142] have been published

for the aluminum and chromium potash alums and for aluminum and iron ammonium alums. On the basis of these results a structure was assigned which placed four sulfate sulfur atoms in a group separated from oxygen atoms. This was chemically so improbable that a more reasonable structure has been sought from these same data. This latter investigation [143] is held to show that the recorded experimental data require T_h^2 as the corresponding space-group. An atomic arrangement is discussed which is a special case of this space-group and which besides accounting for the experiments has the added advantage of not doing violence to chemical conceptions. In other cases, of which rutile is typical, a spectrometer study as ordinarily carried out has so clearly shown its insufficiency as a means of recording correct diffraction data that more conclusive experiments are to be desired.

The lengths of the sides of the unit cubes containing four chemical molecules for the alums [142] measured are:

Crystal.	$a \times 10^{-8}$ cm.
$KAl(SO_4)_2 \cdot 12H_2O$	12.08
$NH_4Al(SO_4)_2 \cdot 12H_2O$	12.00
$NH_4Fe(SO_4)_2 \cdot 12H_2O$	12.14
$KCr(SO_4)_2 \cdot 12H_2O$	11.93

BORATES, ALUMINATES, FERRITES, ETC.

Tourmaline.[144]—Much Laue photographic and spectrographic data have been published upon crystals of tourmaline. From these, notably the latter, an attempt has been made to establish the corresponding space-group. No effort has been made to determine the atomic arrangement more intimately.

Spinel Group, $R''R_2'''O_4$ where R'' and R''' are various di- and trivalent metal elements. Some data have been published upon three members of this group: Magnetite (Fe_3O_4), spinel ($MgAl_2O_4$) and gahnite ($ZnAl_2O_4$). Spectrometric measurements,[145] in the last case only upon a single face, have been recorded for all three. Laue photographs [146] have been taken of the first two. Powder photographs [70] of magnetite, which were not used for structure determination, and of Co_3O_4, which is said to give spectra similar to Fe_3O_4 and is probably isomorphous with it, are also available. Neither of the determinations of crystal structure, the one based upon spectrometric observations alone,[145] the other developed from a detailed consideration of Laue photographs, has been carried out in a conclusive fashion.

The structure deduced by both is, however, the same. Especially because of the excellent character of the second investigation which used space-group results for the purposes of description but apparently not as a direct means of analysis, and because of the agreement of these structures with the more completely studied potassium zinc cyanide and silver molybdate, the correct atomic arrangement for these spinels has undoubtedly been selected. Since this structure is the same as that of the previously discussed potassium zinc cyanide, its atomic arrangement will be described in terms of that crystal. The coördinate positions of the trivalent metal of the spinel are the same as those of potassium, the divalent atoms have the same positions in the two cases and the oxygen atoms have the same set of positions that is occupied by either the carbon or the nitrogen atoms (Fig. 18). With the use of Laue photographic data the value of the parameter u for oxygen could be most accurately determined in the magnesium aluminum spinel as very close to 3/8, the best fit for scattering assumptions used being 0.384. In magnetite the oxygen atoms also lie very close to 3/8, but because of the greater scattering power of the iron atoms an appreciable variation from precisely this position could not well be detected. From the spectrometric measurements the length of the side of the unit cube of magnetite which contains eight chemical molecules has been determined as: $a = 8.30$A.U. Dimensions have not been stated for the other two crystals (though, of course, values can be calculated from the density).

Chalcopyrite, $CuFeS_2$.—Spectrometric measurements [147] of this tetragonal crystal have been made and a structure suggested to account for them. They are so inadequate for the establishing of its correct atomic arrangement, however, that until further work has been carried out this structure cannot be considered as known.

PHOSPHATES, ETC.

Xenotime,[80, 112] YPO_4.—This is the only member of the group which has been subjected to any X-ray examination. The study of this crystal was carried out at the same time and in the same manner as those of rutile and the other tetragonal minerals which it somewhat resembles; hence it is subject to the same insufficiencies as they.

ORGANIC COMPOUNDS.

The crystal structure of no organic compound has been adequately and completely determined. Only one [148] of the few that have been measured, sodium acid acetate, is cubic and has been subjected to the detailed treatment that the use of the results of the theory of space-groups makes possible. Besides this relatively simple substance, the crystal structure of which nevertheless was too complicated for complete elucidation, some powder photographic [78] and spectrometric [149] observations have been made upon numerous organic substances. As determinations of crystal structure the treatment of these latter data are valueless. A determination of the shape and dimensions of the molecule of liquid benzene [150] has been published; but is not in any sense unique.

Sodium Acid Acetate, $NaH(C_2H_3O_2)_2$.—With the aid of spectrographic and Laue photographic data, it is shown [148] that the unit cube of this crystal must contain twenty-four chemical molecules, and that the corresponding space-group is probably T_h^7. This number of atoms within the unit is so great that even with the knowledge of the appropriate space-group no definite picture of the manner of arrangement of these atoms can be obtained. The length of the side of this large unit is given as 15.98A.U.

The exceedingly complicated character of this crystal may serve as a warning of the difficulties that will be encountered when organic compounds are studied. It clearly shows, if any indications aside from the general insufficiency of the methods used were required, of how little use as determinations of correct atomic arrangement are the other studies [78, 149] to which reference has just been made. For this reason they can receive no detailed treatment at this point.

SUMMARY OF THE CRYSTAL STRUCTURES OF NITRATES, CARBONATES, ETC.

The results of crystal structure determinations of the nitrates and crystals following them in the classification of Table I are summarized in Table V.

TABLE V.

The Crystal Structures of the Nitrates, Carbonates, Etc.

Compound.	System of Symmetry.	Type of Arrangement.	Text Figure of Unit Cell.	Dimensions of Unit Cell.	Class of Determination.
$NaNO_3$	Hexagonal	Calcite	Fig. 20	$a = 6.32$; $\alpha = 48° 6'$	IB
$NaClO_3$	Cubic	$NaClO_3$	.....	$a = 6.56, 6.55$	IA
$NaBrO_3$	Cubic	$NaClO_3$	.....	$a = 6.74, 6.71$	IA
$Ca(NO_3)_2$	Cubic	Pyrite type	.....	$a = 7.60$	IIA
$Sr(NO_3)_2$	Cubic	Pyrite type	.....	$a = 7.81$	IB
$Ba(NO_3)_2$	Cubic	Pyrite type	.....	$a = 8.11$	IB
$Pb(NO_3)_2$	Cubic	Pyrite type	.....	$a = 7.84$	IB
$Zn(BrO_3)_2.6H_2O$	Cubic	$Zn(BrO_3)_2.6H_2O$	.....	$a = 10.31$	IA
$Ni(NO_3)_2.6NH_3$	Cubic	$Zn(BrO_3)_2.6H_2O$	.....	$a = 10.96$	IA
$CaCO_3$	Hexagonal	Calcite	Fig. 20	$a = 6.36$; $\alpha = 46° 6'$	IA
$MnCO_3$	Hexagonal	Calcite	Fig. 20	$a = 5.84$; $\alpha = 47° 45'$	IA
$FeCO_3$	Hexagonal	Calcite	Fig. 20	$a = 5.82$; $\alpha = 47° 45'$	IA
Olivine	Orthorhombic	.....	.....	.....	Incomplete
Garnet	Cubic	.....	.....	.....	Uncertain
Alkali sulfates	Orthorhombic	.....	.....	.....	IIB
Ag_2MoO_4	Cubic	$K_2Zn(CN)_4$	Fig. 18	$a = 9.26$	IA
$CaWO_4$	Tetragonal	.....	.....	.....	IIB
$PbMoO_4$	Tetragonal	.....	.....	.....	IIB
$KAl(SO_4)_2.12H_2O$	Cubic	.....	.....	$a = 12.08$	III
$KCr(SO_4)_2.12H_2O$	Cubic	.....	.....	$a = 11.93$	III
$NH_4Al(SO_4)_2.12H_2O$	Cubic	.....	.....	$a = 12.00$	III
$NH_4Fe(SO_4)_2.12H_2O$	Cubic	.....	.....	$a = 12.14$	III
Fe_3O_4	Cubic	$K_2Zn(CN)_4$	Fig. 18	$a = 8.30$	IB
$MgAl_2O_4$	Cubic	$K_2Zn(CN)_4$	Fig. 18	.....	IB
$CuFeS_2$	Tetragonal	.....	.....	.....	IIB
YPO_4	Tetragonal	.....	.....	.....	IIB
$NaH(C_2H_3O_2)_2$	Cubic	.....	.....	$a = 15.98$	IA (Inc.)

CONCLUSIONS.

The known failures to assign correct crystal structures supply confirmation of the point of view expressed in the introduction to this paper. The following are some well-established examples:

Graphite.—Powder data have given two different structures to this crystalline arrangement.

Antimony and Bismuth.—Four atomic arrangements have been given to these similar metals through data drawn from spectrometric and powder data.

Carborundum.—Spectrometric observations have been used to assign a distorted "zinc sulfide arrangement" to the atoms of CSi; later these same data were considered to yield this arrangement undistorted. Powder data have been thought to show a structure which is a mixture of hexagonal and cubic arrangements within the same crystal. Recent crystallographic and Laue photographic measurements, however, appear to show that three different forms of CSi crystals can exist together and that the structure of at least one of them is very complicated. From such data as are available it appears that the crystals studied with the spectrometer are the same as those investigated with Laue photographs; the results of the two investigations, nevertheless, are different. It is not apparent which or how many of the forms were subjected to powder measurement. The insufficiency of these individual methods taken alone is evident.

Ice.—Three different structures have come out of two experimental studies upon ice, one spectrographic and the other powder photographic. A theoretically well-worked treatment of Laue photographs has led to still a different arrangement. Because of the uncertainty concerning the individual character of the specimens used in the last-mentioned investigation, it cannot now be considered as any more certain than the others. Even the dimensions of unit cells assigned by these different studies are not related to one another.

Beryllium Oxide.—By ascribing certain of the lines appearing in powder photographs to impurities in the sample, a cubic structure has been given to this hexagonal crystal.

Zinc Oxide.—Spectrometric observations were believed to have established a structure for this crystal. A subsequent application of space-group considerations, however, brought to light

another simple atomic arrangement which was in equally good agreement with the existing data and in almost as good agreement with any data that can now be obtained.

Silica.—Two different structures have been assigned to quartz upon the basis of spectrometer measurements.

Rutile.—The spectrometric data as ordinarily collected, and consequently the structures that have been assigned to this crystal by the two investigations that have been made, are different.

Silver Iodide.—Two separate determinations with powder data have given this hexagonal crystal a cubic structure. A careful study with both Laue photographic and powder data has since shown that the principal powder lines are in close agreement with positions for a structure with cubic symmetry. Silver iodide is a clear case against crystal structure determinations based upon unaided powder photographic data.

Tetramethylammonium Iodide.—On the basis of spectrometer measurements a structure was assigned to this crystal which did not group the four methyl carbon atoms about the corresponding nitrogen atom. This appeared so improbable that upon search another structure was found which was chemically more probable and in equally good agreement with the experimental data.

Sodium Chlorate.—Two determinations of the structure of sodium chlorate, one with powder data the other from spectrometric measurements, give the same kind of arrangement but with different positions for the oxygen atoms. The powder data, however, are in equally good agreement with both sets of oxygen positions; the spectrometric results disagree with the arrangement based simply upon powder data only for those planes which cannot be separated in powder experiments.

Potash Alum.—On the basis of spectrometer data alone an atomic arrangement was assigned to this crystal which grouped four sulfur atoms together. This structure was in such disagreement with chemical notions that another grouping of atoms was found which was in equally good agreement with the experimental data.

Potassium Triiodide.—On the basis of measurements made by a modified spectrometer technic, this monoclinic crystal has received a cubic structure.

In the face of these illustrations no further argument is required to show the practical necessity of adopting methods of

crystal structure study that make use of space-group results and of using to its full the crystallographic information concerning symmetry. These failures are amongst the relatively complicated crystals which either have a symmetry lower than cubic or are of considerable chemical complexity. This need becomes even more strikingly apparent when it is noted that few of these complicated crystals have been studied and when it is realized that the restricted field of simple cubic crystals has been quite thoroughly explored. The failure of the data that can be obtained by some of the methods of producing X-ray effects used alone must be convincing argument for taking a wider view towards diffraction experiments. Such an attitude, seeing in no one method an answer to all questions that may arise, rather employs in any particular case that procedure which is calculated to yield the most usable data.

BIBLIOGRAPHY.

[1] DANA: "System of Mineralogy."
[2] P. GROTH: "Chemische Krystallographie," Leipzig, 1906.
[3] A. W. HULL: *Phys. Rev.*, **10**, 661 (1917).
[4] J. M. BIJVOET and A. KARSSEN: *Proc. Roy. Acad. Sci. Amsterdam*, **23**, No. 9 (1922).
[5] L. W. MCKEEHAN: *Proc. Nat. Acad. Sci.*, **8**, 254 (1922).
[6] W. L. BRAGG: *Phil. Mag.*, **28**, 355 (1914).
[7] L. VEGARD: *Phil. Mag.*, **31**, 83 (1916).
[8] H. KAHLER: *Phys. Rev.*, **18**, 210 (1921).
[9] L. VEGARD: *Phil. Mag.*, **32**, 65 (1916).
[10] L. W. MCKEEHAN: *Proc. Nat. Acad. Sci.*, **8**, 270 (1922).
[11] A. W. HULL: *Proc. Nat. Acad. Sci.*, **3**, 470 (1917).
[12] H. BOHLIN: *Ann. d. Physik*, **61**, 421 (1920).
[13] A. W. HULL: *Phys. Rev.*, **17**, 42 (1921).
[14] P. NIGGLI: "Geometrische Krystallographie des Discontinuums," Leipzig. 1919.
[15] L. W. MCKEEHAN and P. P. CIOFFI: *Phys. Rev.*, **19**, 444 (1922).
[16] N. ALSEN and G. AMINOFF: *Geol. För. Förh.*, **44**, 124 (1922).
[17] A. W. HULL: *Phys. Rev.*, **9**, 564 (1917).
[18] P. SCHERRER: *Physikal. Z.*, **19**, 23 (1918).
[19] A. W. HULL: *Phys. Rev.*, **17**, 571 (1921).
[20] W. H. and W. L. BRAGG: *Nature*, **91**, 557 (1913).
[21] P. P. EWALD: *Ber. Bayer. Akad. Wiss.*, p. 325 (1914).
[22] P. DEBYE and P. SCHERRER: *Physikal. Z.*, **17**, 277 (1916).
[23] P. DEBYE and P. SCHERRER: *Physikal. Z.*, **18**, 291 (1917).
[24] A. W. HULL: *Phys. Rev.*, **20**, 113 (1922).
[25] W. GERLACH: *Physikal. Z.*, **23**, 114 (1922).

[26] W. GERLACH: *Physikal. Z.*, **22**, 557 (1921).
[27] A. W. HULL: *Science*, **52**, 227 (1920).
[28] A. W. HULL: *Phys. Rev.*, **18**, 88 (1921).
[29] N. H. KOLKMEIJER: *Verslag. Akad. Wetenschappen*, **31**, 155; *Proc. Roy. Acad. Sci. Amsterdam*, **25**, 125 (1922).
[30] A. J. BIJL and N. H. KOLKMEIJER: *Chem. Weekblad*, **15**, 1264 (1918); *Proc. Roy. Acad. Sci. Amsterdam*, **21**, 405, 494, 501 (1919).
[31] R. W. JAMES and N. TUNSTALL: *Phil. Mag.*, **40**, 233 (1920).
[32] A. OGG: *Phil. Mag.*, **42**, 163 (1921).
[33] R. W. JAMES: *Phil. Mag.*, **42**, 193 (1921).
[34] W. H. and W. L. BRAGG: "X-rays and Crystal Structure," London, 1918, p. 227.
[35] W. H. BRAGG: *Proc. Roy. Soc. A.*, **89**, 575 (1914).
[36] P. STOLL: *Arch. Sci. Phys. Nat.*, **3**, 546 (1921).
[37] P. DEBYE: *Physikal. Z.*, **18**, 483 (1917).
[38] A. W. HULL: *Phys. Rev.*, **9**, 84 (1917).
[39] A. W. HULL: *Phys. Rev.*, **14**, 540 (1919).
[40] A. WESTGREN: *J. Iron and Steel Inst.*, No. 11 (1921).
[41] A. WESTGREN and A. E. LINDH: *Zeitsch. physikal. Chem.*, **98**, 181 (1921).
[42] L. W. MCKEEHAN: *Phys. Rev.*, **20**, 82 (1922).
[43] M. R. ANDREWS: *Phys. Rev.*, **18**, 245 (1921).
[44] A. WESTGREN and G. PHRAGMEN: *J. Iron and Steel Inst.*, 1922.
[45] L. W. MCKEEHAN: *Phys. Rev.*, **19**, 537 (1922).
[46] G. AMINOFF and G. PHRAGMEN: *Zeitsch. f. Krystal.*, **56**, 510 (1921).
[47] H. BAUMHAUER: *Zeitsch. f. Krystal.*, **50**, 33 (1912); **55**, 249 (1915–20).
[48] F. v. HAUER and P. KOLLER: *Zeitsch. f. Krystal.*, **50**, 260 (1915–20).
[49] C. L. BURDICK and E. A. OWEN: *J. Am. Chem. Soc.*, **40**, 1749 (1918).
[50] A. W. HULL: *Phys. Rev.*, **15**, 545 (1920).
[51] H. ESPIG: *Abh. Sächs. Akad. Wiss. Leipzig*, (Math. Phys. Klasse) **38**, 53 (1921).
[52] A. E. H. TUTTON: "Crystallography and Practical Crystal Measurement," Vol. i, p. 689, London, 1922.
[53] A. W. HULL, *Phys. Rev.*, **13**, 292 (1919).
[54] F. RINNE: *Ber. Verh. K. Sächs, Gesell. Wiss. Math. Nat.*, **69**, 57 (1917).
[55] R. GROSS: *Centr. Min. Geol.*, 1919, p. 201.
[56] A. ST. JOHN: *Proc. Nat. Acad. Sci.*, **4**, 193 (1918).
[57] D. M. DENNISON: *Phys. Rev.*, **17**, 20 (1921); *Chem. News.*, **122**, 54 (1921).
[58] W. H. BRAGG: *Proc. Phys. Soc. London*, **34**, 98 (1922).
[59] W. H. and W. L. BRAGG: "X-rays and Crystal Structure," London, 1918, p. 155.
[60] W. P. DAVEY: *Phys. Rev.*, **19**, 248 (1922).
[61] P. NIGGLI: *Zeitsch. f. Krystal.*, **57**, 253 (1922).
[62] R. W. G. WYCKOFF: *Am. J. Sci.*, **3**, 184 (1922).
[63] W. GERLACH: *Zeitsch. f. Physik*, **9**, 184 (1922).
[64] W. L. BRAGG: *Proc. Roy. Soc. A.*, **89**, 248 (1913).
[65] A. W. HULL: *Proc. Am. Inst. Elec. Engin.*, **38**, 1171 (1919).
[66] W. P. DAVEY and E. O. HOFFMAN: *Phys. Rev.*, **15**, 333 (1920).

[87] W. GERLACH and O. PAULI: *Zeitsch. f. Physik*, **7**, 116 (1921).
[88] R. W. G. WYCKOFF: *Am. J. Sci.*, **1**, 138 (1921).
[89] E. SCHIEBOLD: *Zeitsch. f. Krystal.*, **56**, 430 (1921).
[70] J. A. HEDVALL: *Arkiv. Kemi Mineral. Geol.*, **8**, No. 11; *Zeitsch. anorg. Chem.*, **120**, 327 (1922).
[71] W. P. DAVEY: *Phys. Rev.*, **17**, 402 (1921).
[72] W. L. BRAGG: *Phil. Mag.*, **39**, 647 (1920).
[73] E. S. FEDEROV: *Bull. Acad. Sci. Petrograd*, p. 359 (1916).
[74] G. AMINOFF: *Zeitsch. f. Krystal.*, **56**, 495 (1921).
[75] G. AMINOFF: *Zeitsch. f. Krystal.*, **57**, 204 (1922).
[76] P. SCHERRER: *Zeitsch. f. Krystal.*, **57**, 186 (1922).
[77] J. BECKENKAMP: *Zeitsch. anorg. Chem.*, **110**, 290 (1920).
[78] K. BECKER and W. JANCKE: *Zeitsch. physikal. Chem.*, **99**, 242 (1921).
[79] C. M. WILLIAMS: *Proc. Roy. Soc.*, **93**, 418 (1917).
[80] L. VEGARD: *Phil. Mag.*, **32**, 505 (1916).
[81] W. H. and W. L. BRAGG: "X-rays and Crystal Structure," London, 1918, p. 169.
[82] G. AMINOFF: *Geol. För. Förh.*, **41**, 407 (1919).
[83] G. AMINOFF: *Zeitsch. f. Krystal.*, **56**, 506 (1921).
[84] R. W. G. WYCKOFF: *Am. J. Sci.*, **2**, 239 (1921).
[85] W. H. and W. L. BRAGG: *Proc. Roy. Soc. A.*, **89**, 276 (1914).
[86] W. L. BRAGG: *Proc. Roy. Soc. A.*, **89**, 468 (1914).
[87] R. W. G. WYCKOFF: "The Analytical Expression of the Results of the Theory of Space-groups," Carnegie Inst. Wash. Pub. 318 (1922).
[88] M. K. SLATTERY: *Phys. Rev.*, **20**, 84 (1922).
[89] W. H. and W. L. BRAGG: "X-rays and Crystal Structure," London, 1918, p. 173.
[90] P. P. EWALD: *Physikal. Z.*, **15**, 399 (1914).
[91] P. P. EWALD and W. FRIEDRICH: *Ann. d. Physik.*, **44**, 1183 (1914).
[92] P. P. EWALD: *Ann. d. Physik*, **44**, 257 (1914).
[93] M. MECHLING: *Abh. Sächs. Akad. Wiss. Leipzig*, **38**, 37 (1921).
[94] J. M. BIJVOET and A. KARSSEN: *Proc. Roy. Acad. Sci. Amsterdam*, **25**, Nos. 1 and 2 (1922).
[95] R. W. G. WYCKOFF: *J. Wash. Acad. Sci.*, **11**, 429 (1921).
[96] EUGEN POSNJAK and R. W. G. WYCKOFF: *J. Wash. Acad. Sci.*, **12**, 248 (1922).
[97] W. P. DAVEY: *Phys. Rev.*, **18**, 102 (1921).
[98] R. GLOCKER: *Ann. d. Physik*, **47**, 377 (1915); *Physikal. Z.*, **15**, 401 (1914).
[99] W. H. BRAGG: *Proc. Roy. Soc. A.*, **89**, 246 (1913).
[100] M. SIEGBAHN: *Phil. Mag.*, **37**, 601 (1919), etc.
[101] R. G. DICKINSON, *J. Am. Chem. Soc.*, **44**, 2404 (1922).
[102] W. P. DAVEY: *Phys. Rev.*, **19**, 538 (1922).
[103] W. P. DAVEY and F. G. WICK: *Phys. Rev.*, **17**, 403 (1921).
[104] R. W. G. WYCKOFF and EUGEN POSNJAK: *J. Am. Chem. Soc.*, **44**, 30 (1922).
[105] G. AMINOFF: *Geol. För. Förh.*, **44**, 444 (1922).
[106] G. AMINOFF: *Zeitsch. f. Krystal.*, **57**, 180 (1922).
[107] R. B. WILSEY: *Phil. Mag.*, **42**, 262 (1921).

[108] R. W. G. WYCKOFF: *Am. J. Sci.*, **4**, 469 (1922).
[109] G. BARTLETT and I. LANGMUIR: *J. Am. Chem. Soc.*, **43**, 84 (1921).
[110] R. W. G. WYCKOFF: *Am. J. Sci.*, **3**, 177 (1922).
[111] P. NIGGLI: *Zeitsch. f. Krystal.*, **56**, 212 (1921–2).
[112] L. VEGARD: *Phil. Mag.*, **33**, 395 (1917).
[113] R. G. DICKINSON: *J. Am. Chem. Soc.*, **44**, 1489 (1922).
[114] P. A. COOPER: *Nature*, **107**, 745 (1921).
[115] R. M. BOZORTH: *J. Am. Chem. Soc.*, **44**, 317 (1922).
[116] R. M. BOZORTH: *J. Am. Chem. Soc.*, **44**, 2232 (1922).
[117] R. W. G. WYCKOFF: *Am. J. Sci.*, **5**, 15 (1923).
[118] R. W. G. WYCKOFF: *J. Am. Chem. Soc.*, **44**, 1239 (1922).
[119] P. SCHERRER and P. STOLL: *Zeitsch. anorg. Chem.*, **121**, 319 (1922).
[120] R. W. G. WYCKOFF: *J. Am. Chem. Soc.*, **42**, 1100 (1920).
[121] G. L. CLARK and W. DUANE: *Phys. Rev.*, **20**, 85 (1922).
[122] R. G. DICKINSON: *J. Am. Chem. Soc.*, **44**, 774 (1922).
[123] R. W. G. WYCKOFF and E. POSNJAK: *J. Am. Chem. Soc.*, **43**, 2292 (1921).
[124] R. G. DICKINSON: *J. Am. Chem. Soc.*, **44**, 276 (1922).
[125] R. M. BOZORTH: *J. Am. Chem. Soc.*, **44**, 1066 (1922).
[126] R. W. G. WYCKOFF: *Phys. Rev.*; **16**, 149 (1920).
[127] N. H. KOLKMEIJER, J. M. BIJVOET and A. KARSSEN: *Proc. Roy. Acad. Sci. Amsterdam*, **23**, 644 (1921).
[128] R. G. DICKINSON and E. A. GOODHUE: *J. Am. Chem. Soc.*, **43**, 2045 (1921).
[129] S. NISHIKAWA and K. HUDINUKI: *Proc. Math. Phys. Soc. Tokyo*, **9**, 197 (1917).
[130] L. VEGARD: *Zeitsch. f. Physik*, **9**, 395 (1922).
[131] R. W. G. WYCKOFF: *Am. J. Sci.*, **4**, 188 (1922).
[132] R. W. G. WYCKOFF: *J. Am. Chem. Soc.*, **44**, 1260 (1922).
[133] W. H. BRAGG: *Phil. Trans. Roy. Soc.*, **215**, 253 (1915).
[134] E. SCHIEBOLD: *Abh. Sächs. Akad. Wiss. Leipzig*, **36**, 65 (1919).
[135] R. W. G. WYCKOFF: *Am. J. Sci.*, **50**, 317 (1920).
[136] C. BERNDT: *Abh. Sächs. Akad. Wiss. Leipzig*, **38**, 1 (1921).
[137] S. NISHIKAWA: *Proc. Math. Phys. Soc. Tokyo*, **9**, 194 (1917).
[138] A. OGG and F. L. HOPWOOD: *Phil. Mag.*, **32**, 518 (1916).
[139] R. W. G. WYCKOFF: *J. Am. Chem. Soc.*, **44**, 1994 (1922).
[140] R. G. DICKINSON: *J. Am. Chem. Soc.*, **42**, 85 (1920).
[141] T. TERADA: *Proc. Math. Phys. Soc. Tokyo*, **7**, 290 (1914).
[142] L. VEGARD and H. SCHJELDERUP: *Ann. d. Physik*, **54**, 146 (1918).
[143] P. NIGGLI: *Physikal. Z.*, **19**, 225 (1918).
[144] C. KULASZEWSKI: *Abh. Sächs. Akad. Wiss. Leipzig*, **38**, 81 (1921).
[145] W. H. BRAGG: *Phil. Mag.*, **30**, 305 (1915); *Nature*, **95**, 561 (1915).
[146] S. NISHIKAWA: *Proc. Math. Phys. Soc. Tokyo*, **8**, 199 (1915).
[147] C. L. BURDICK and J. H. ELLIS: *Proc. Nat. Acad. Sci.*, **3**, 644 (1917); *J. Am. Chem. Soc.*, **39**, 2518 (1917).
[148] R. W. G. WYCKOFF: *Am. J. Sci.*, **4**, 193 (1922).
[149] W. H. BRAGG: *Proc. Phys. Soc. London*, **34**, 33 (1921).
[150] P. DEBYE and P. SCHERRER: *Nachr. Ges. Wiss. Göttingen*, 1916.
[151] L. W. MCKEEHAN: *Phys. Rev.*, **20**, 424 (1922).

THE STABILITY OF ATOM NUCLEI, THE SEPARATION OF ISOTOPES, AND THE WHOLE NUMBER RULE.

SUPPLEMENTARY NOTE.

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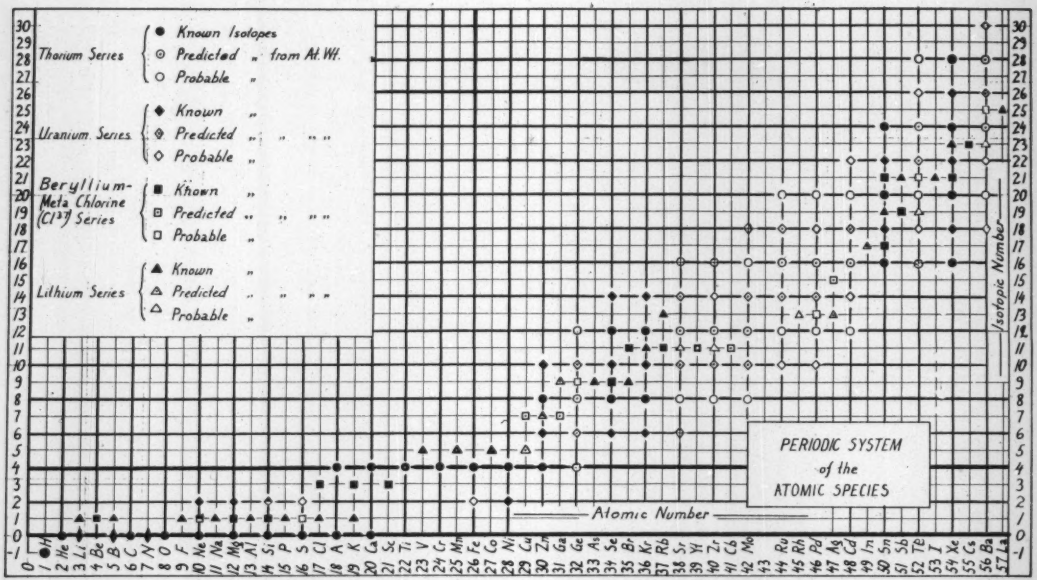
§3. ATOMIC SPECIES OF ODD ATOMIC WEIGHT.

THE earlier sections of the paper have shown that most of the isotopes of elements of odd atomic number have odd atomic weights and isotopic numbers, while for elements of even atomic number these numbers are even. No general relationship was given in regard to the existence of the much more rare atomic species: those which have an odd atomic weight and isotopic number, associated with an even atomic number. Not only is the number of such atomic species small, but they are also represented in nature by only a comparatively small number of atoms. An empirical relation concerning such isotopic species is made evident by Fig. 3, which plots the atomic number on the X-axis and the isotopic number on the Y-axis. The atomic species of the class in question and below atomic number 38, which have thus far been discovered are: [124] $_{4}Be^{9}$, $_{12}Mg^{25}$, $_{14}Si^{29}$, $_{34}Se^{77}$, and $_{36}Kr^{83}$. It will be seen that on the plot *each of these atomic species is between two atomic species of the same isotopic number and next higher and next lower atomic numbers.* Thus any isotope of odd atomic weight in an element of even atomic number has the same isotopic number as the adjacent isotopes of elements of odd atomic number, and lies on the same neutronal line (horizontal line in the figure) between them. While not every position of this type is occupied by an atomic species, it is of interest that every atomic species of this class falls in just such a position.

When the atomic number becomes higher the condition for the existence of odd-numbered isotopes of even-numbered elements seems to become less exacting, in that these isotopes occur when there is an atomic species of the same isotopic number existent

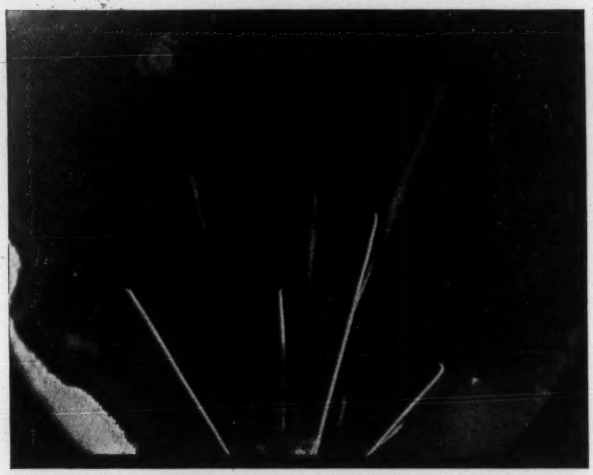
[124] Here the atomic number is written first, the subscript is the isotopic number and the superscript the atomic weight.

FIG. 43.


Thorium Series
Known Isotopes
Predicted „ from At. Wt.
Probable „
Uranium Series
Known „
Predicted „ „ „ „
Probable „
Beryllium-Meta Chlorine (Cl37) Series
Known „
Predicted „ „ „ „
Probable „
Lithium Series
Known „
Predicted „ „ „ „
Probable „
PERIODIC SYSTEM of the ATOMIC SPECIES
Atomic Number
Isotopic Number

in only one, instead of in both of the elements of next higher and lower atomic number. Thus xenon has two isotopes of the class under discussion 54 Xe^{129}_{21}, and 54 $Xe^{131}_{?}$. The former of these has the same isotopic number as 53 $I^{129}_{?}$, and the latter, as 55 Cs^{13}_{23}. Thus the occurrence of odd atomic weight isotopes of elements of even atomic number is plainly related to the exist-

FIG. 44.



Straight tracks with curves at end.

ence of odd atomic weight isotopes in adjacent elements of odd atomic number.

The relation given above indicates the probability of the existence of an isotope of zinc of isotopic number 7, since both of the adjacent elements, copper and gallium, have chemical atomic weights which indicate the existence of an isotope of copper, and one of gallium, of this atomic number. The work of Dempster also shows the high probability that such a zinc isotope exists, since a peak in this position occurs in some of his ionization curves.

Fig. 43 shows that in certain regions every successive corner

along a neutronal line is filled, that is, every possible atomic species exists. This is true along certain lines of odd isotopic number, as istopic number 1 from sodium to phosphorus. It is probable that elements 31, 32, 33, 34, and 35, all have isotopes of isotopic number 9, and elements 50, 51, 52, 53, and 54, of isotopic number 21. It will be seen that for atomic numbers greater than 28 there are *commonly four or more levels on which atomic species*

FIG. 45.

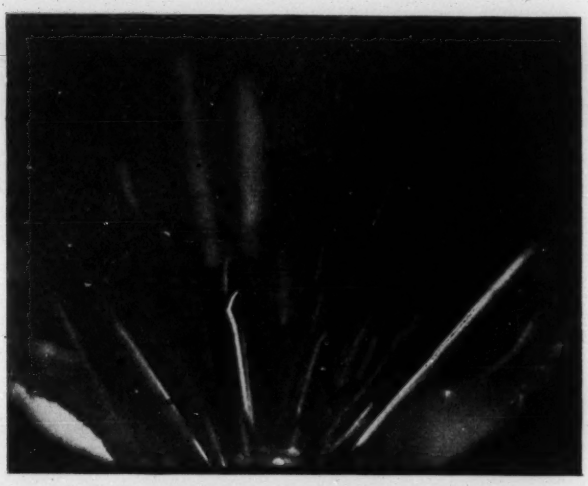
Straight tracks with one curved at end.

of even atomic and isotopic number occur, but that usually there is only one level for species of odd isotopic number, except that there are often two levels for elements which give a rise from one level to another. Thus bromine contains the atomic species of *highest atomic number* which has isotopic number 9, and the atomic species of *lowest atomic number* in which the isotopic number is 11. There are undoubtedly some isotopes which rise above the common level, as is the case with rubidium of isotopic number

13. *On the whole the atomic species of odd atomic* WEIGHT (odd isotopic number) *are arranged in the plot in a series of single steps rising from left to right,* but occasionally one step lies above another for short distance.

It is apparent that from helium to lithium of atomic weight 6, and from boron of atomic weight 10 to oxygen, all of which atomic species have an isotopic number zero, the increment in

FIG. 46.



Left turn followed by right turn and a curve.

the composition of the nucleus for an increase of 1 in the atomic number is represented by the formula p_2e, which is just one-half of an alpha particle (p_4e_2). Now the above paragraphs of this section have made it evident that while the increment between nuclei of the same isotopic number is almost always a whole α-particle when the atomic weight is even, the increment is often only a half α-particle, or the group p_2e, when the atomic weight is odd, though in certain other regions it is a whole alpha particle.

This relation calls to mind a suggestion made by the writer several years ago, before this last relation was discovered, to the effect that the group p_2e may be of fundamental importance in connection with atom building, especially since the formula of any nucleus may be represented as $(p_2e)_M(pe)_n$, in which M is the atomic and n is the isotopic number. For species of zero isotopic number this reduces to $(p_2e)_M$. It might be supposed that the

FIG. 47.

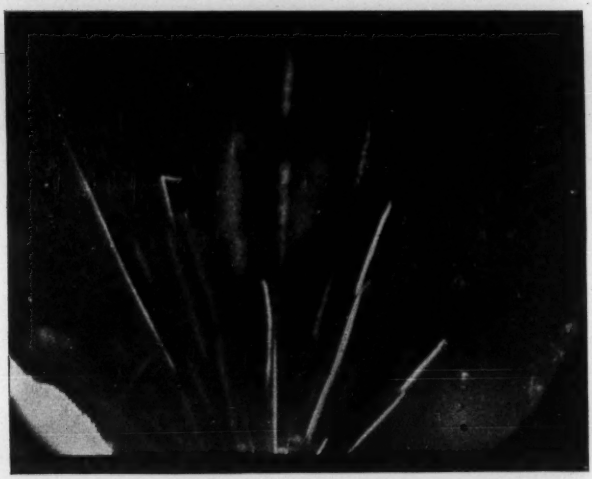
Straight track and track with sharp bend.

p_2e group is very stable with respect to disintegration, but that in general it does not exist alone, since it readily unites with itself to form a double p_2e group, or alpha particle, or less often attaches itself to a heavier nucleus. If this is the case, the relation given above would indicate that it attaches itself to a heavy nucleus most easily when the weight of the latter is an odd number. It will be recognized that such suggestions are much more hypothetical than the rest of the material presented in the paper.

84. DISCOVERY OF ISOTOPES PREDICTED IN EARLIER SECTIONS.

In the chart [125] of the General System of Isotopes (Fig. 21 between pages 654 and 655)[126] numerous predicted and probable isotopes were listed. It is of interest that since the chart was published the isotopes of antimony and selenium have been detected by Aston, with the result that *for both these elements exactly the same isotopes were found as were listed in the chart.* In the

FIG. 48.



Sharp bend in alpha ray track.

discussion of the chart it was pointed out that no specification was made of the rarely occurring isotopes of odd atomic weight and even atomic number. However, the relation given in the preceding section (Section 83) indicates that the odd-numbered isotope of selenium should be of an isotopic number 9, which corresponds to what was found.

[125] Errors in Fig. 21, Element 7 is specified as H, though it is N, and 52 as Tl, though it is Te.

[126] This JOURNAL, Vol. 194, Nov., 1922.

85. RECENT EXPERIMENTAL WORK ON ISOTOPES.[127]

The steel apparatus used by Harkins and Madorsky for the separation of the isotopes of mercury without other cooling than that produced by ice, has given an atomic weight difference of 0.1, while Mulliken in this laboratory has secured the same degree of separation with end samples 22 c.c. each for the light and heavy fractions. In general most of the samples have a

FIG. 49.

Sharp bend in alpha ray track.

volume of 50 c.c. each. The volume of the end fractions is many times larger than that obtained in any previous investigation, and *small* samples of 0.15 unit difference in atomic weight could be obtained from them in about two days of work.

[127] A note in *Nature* of July 14, 1921, states that Bronsted and Hevesy have applied the method of vaporization to hydrogen chloride and have secured a slight separation equal to 0.027 unit of atomic weight.

86. PHOTOGRAPHS OF THE TRACKS OF ALPHA PARTICLES IN GASES, BY R. W. RYAN AND W. D. HARKINS.

A few photographs of alpha ray tracks, as obtained in the preliminary work done in this laboratory, were presented in an earlier section of the paper. About ten thousand photographs have now been obtained, and an optical device has been constructed so that two views at right angles can be taken on the

FIG. 50.

Short stub due to nucleus of nitrogen atom and long spur due to alpha particle.

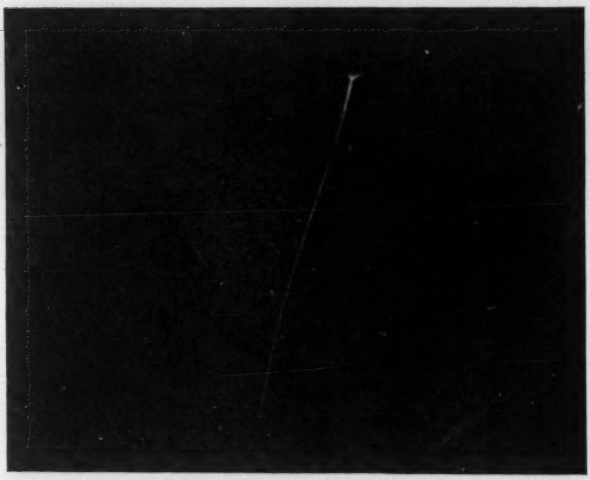
same film. If two motion-picture machines are used at right angles the photographs may be secured upon different films, which gives a brighter image, but has the disadvantage of being twice as costly with respect to film. A large apparatus, which will allow the photographic reproduction of tracks 15 cm. long, is practically ready for use.

It was noted in an earlier photograph that a track is commonly invisible in a region through which another track has just passed, due to the removal of the supersaturation of the

water vapor, as is shown very markedly in Fig. 39*a*. On this account it is not advisable to use a source which gives too many alpha particles.

Fig. 44 shows four practically straight tracks with curved ends. The curvature at the end is due to the low velocity of the alpha particles at the end of the track, since this gives time for the adjacent atom nuclei to noticeably repel the alpha particles

FIG. 51.

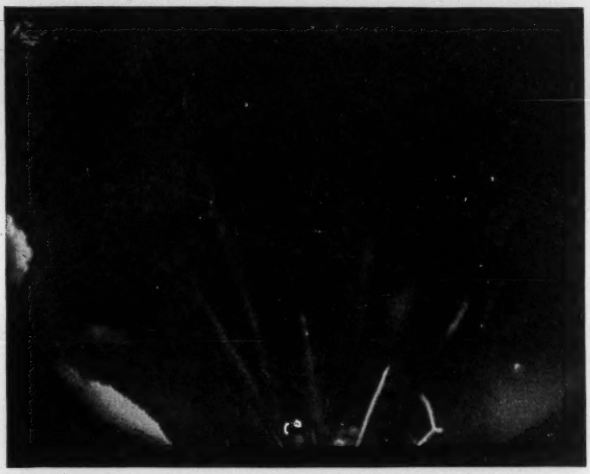


Alpha ray track with double curvature.

and cause what is known as straggling. In Fig. 45 the curvature is particularly great at the end of one of the tracks. A bend toward the left and a later sharp turn to the right, evidently due to a penetration of the alpha particle close to two different nitrogen or oxygen nuclei, may be seen in Fig. 46. The end of this track is evidently influenced by a number of nuclear approaches, since it exhibits a marked curvature. A single sharp bend is to be seen in Fig. 47, and a much sharper one in Fig. 48. A right-angle bend (very nearly), with a slight spur due to the

nucleus of a nitrogen atom, is plainly visible in the photograph from which Fig. 49 is taken. A short stub caused by the nucleus of a nitrogen atom and a long spur, curved at the end, representing the path of the alpha particle after the collision, may be seen in Fig. 50. Fig. 51 is interesting in that it shows a double curvature due to the passage of the alpha particle near two different atom nuclei at a considerable distance apart. This track finally

FIG. 52.



Forked track: One fork due to alpha particle and the other to the nucleus of a nitrogen atom.

ends in a fork due to a very near approach to still another atom nucleus. In each of Figs. 52 and 53 is a fork in which the alpha particle goes in one direction, and the nucleus of the bombarded nitrogen atom in another. Fig. 54 gives a much sharper photograph of a track produced earlier. The fork is nearly in the plane of the picture, the alpha particle being turned through about 155°, while the nitrogen nucleus is hurled forward a considerable distance. In a direct impact the velocity of the rebounding alpha

particle is 0.6, that of a bombarded oxygen atom nucleus 0.4 if the velocity of the alpha particle just before impact is taken as 1, and for a nitrogen nucleus the relations are very nearly the same. A slight straggling of the atom nucleus near the end of its path is indicated by the curve at the end of the track (Fig. 54). Figs. 55 and 56 show tracks in which one limb of the fork is almost a direct continuation of the track of the alpha particle,

FIG. 53.

Same relation as FIG. 52.

while the other branches off at a sharp angle. Bose obtained one such track in hydrogen, so these may represent bombardments against hydrogen nuclei, but more photographs will need to be secured before the origin of this form of fork is made known by the present work. Photographs of the tracks at right angles will be taken in considerable number in the near future. A statistical study of the present set of photographs indicates that the nuclei of nitrogen and oxygen atoms are extremely minute. It is hoped

that data on the dimensions of bromine and iodine atom nuclei may be obtained soon.

The photographs show no indication of a disintegration of the bombarded nuclei, so thorium C will be used to give more rapid alpha particles for later work.

Fig. 54 may be used to illustrate what might be expected provided the nucleus which is bombarded by an alpha particle is

FIG. 54.

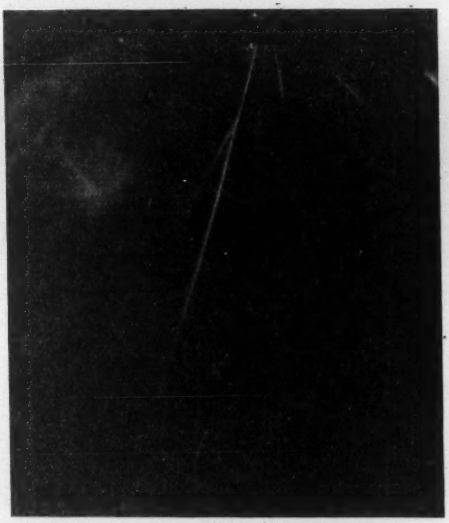
Forked track showing rebound of alpha particle and forward motion of nitrogen atom.

disintegrated. Fig. 54 shows the forward track of the alpha particle, its backward path after the collision, and the forward track of the bombarded nucleus. If the nucleus should disintegrate, then it should give two tracks, one for the heavier, and one for the lighter fragment. In the actual disintegrations thus far obtained the lighter fragment is a hydrogen nucleus of long range. *Thus if a disintegration is secured the original track should split into three branches instead of two.* It is evident, also, that since the disintegrating nucleus would supply energy, there

would not be the simple relation between the angle of incidence and that of reflection exhibited in the figure. Every such disintegration would not be detected, since in some ordinary impacts one of the branches fails to appear.

It is almost certain that the disintegration of nitrogen to give hydrogen could be detected by this means if it were not such a

FIG. 55.



Peculiar fork in track.

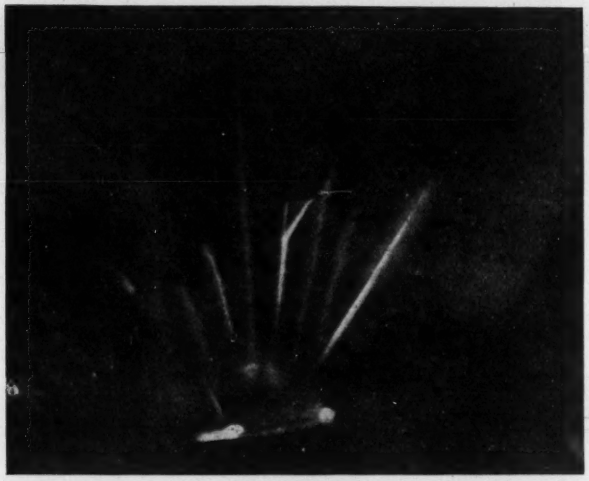
rare event in comparison with the number of tracks involved. The writers are preparing to endeavor to make visible the disintegration of aluminum atoms to give hydrogen, which would be much more simple, being only complicated by the effects of the powerful gamma rays given off by the source.[128]

[128] On account of the large demand for lantern slides and copies of the photographs of alpha ray tracks presented in this paper, the writer has arranged with Mr. R. W. Ryan to supply them at a nominal cost, and directly from the original enlargements, since the photographs suffer considerable loss of clearness in reproduction.

87. SEPARATION OF ISOTOPES BY GASEOUS DIFFUSION THROUGH A MOVING STREAM OF GAS OR VAPOR.

The diffusion of a gas through a moving stream of another gas, suggested by Mulliken and Harkins [129] under the name of "gaseous diffusion with negative mass motion," has been applied to the separation of helium and neon, which are of course not isotopic, by Hertz.[130] He suggests that the method might be of

FIG. 56.



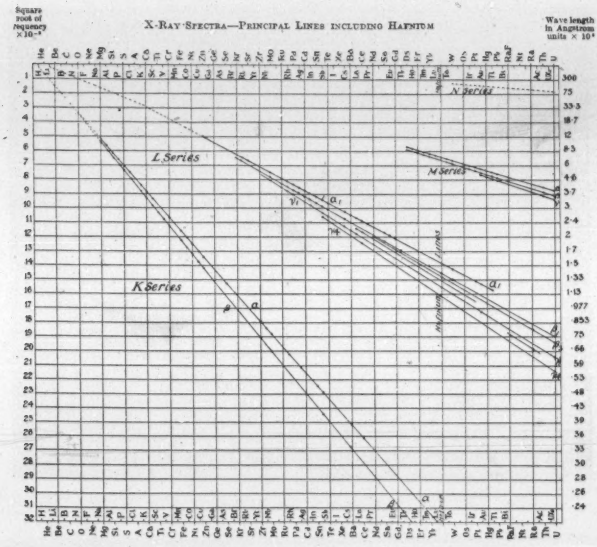
Split track of alpha particle.

use in the separation of isotopes. In order to get a stream of gas which is easily controlled he uses steam, which may be condensed where it is desired that the flow shall stop, and this improvement is what gives the method its promise. Mulliken has studied the principles of this method and some of his principal conclusions will be given below.

[129] *Loc. cit.*, Jan., 1922.
[130] *Phys. Z.*, Dec., 1922, 23, 433.

Hertz considers his method to be greatly superior to other diffusion methods since it may be made to give any degree of separation in a single operation. However, this can be done in any ordinary diffusion process in the case of the *heavy fraction*. The advantage of the method, therefore, lies in extending this advantage to the light fraction also. Hertz has failed to note that the *rate of production* of either fraction decreases exponentially

FIG. 57.



with the degree of separation of the products, which is just the relation found for the ordinary methods. Thus the rate of production of the light fraction decreases logarithmically as the separation in terms of atomic weight increases arithmetically.

If a steady stream of vapor passes from left to right with a velocity v across a narrow space between two parallel planes, being supplied constantly at the plane $X = X$ on the right, and condensed

at an equal rate on the plane $X = O$ on the left, then any gas present at a small partial pressure will distribute itself according to the relation $f = f_o e^{\frac{-vx}{\delta}}$ where δ is the diffusion coefficient of the gas, and f is the fraction of the gas removed at the plane $X = X$, which constitutes the light fraction. This is analogous to the relation found for the distribution of a gas in a gravitational field or under centrifugal force.

The difference in the molecular weight produced between the planes is

$$M = (M_2 - M_1) \frac{\left(1 - \frac{\delta_1}{\delta_2}\right) x_1 x_2}{x_1 + x_2 \frac{\delta_1}{\delta_2}} ln \frac{f_o}{f}.$$

Here x represents the mol fraction. This equation is given approximately [131] by

$$M = aB \ln \frac{f_o}{f}$$

For molecular diffusion the ratio of the diffusion coefficients of two isotopes is $\left(\frac{M_2}{M_1}\right)^{1/2}$, and in gaseous diffusion it is $\left(\frac{M_2}{M_1}\right)^{1/c}$ where c is equal to or greater than 2. The value of a in the above equations is thus equal to or less than 1, since a is the ratio of $1/c$ to $1/2$. From the formulas given in Jeans ("Dynamical Theory of Gases") the above conclusion that c is commonly greater than 2 for gaseous diffusion, may be deduced. The value of a may even approach zero if a gas of high molecular weight, such as mercury, is diffused in a vapor of very low molecular weight, and it should approach the value of one if the molecular weights are reversed, and is probably near ¾ when they are equal. Since in molecular diffusion and in low pressure vaporization the value of a is one, the lower value in gaseous diffusion is detrimental to the method of Hertz, and to any other method of gaseous diffusion, such as the diffusion of hydrogen chloride into air used by Harkins and Broeker, and Harkins and Hayes. However, the success of the work done by these investigators indicates the utility of the method in spite of the above listed disadvantages.

[131] The equation for the residue in ordinary gaseous diffusion is $\Delta M = aBlnC$, where C is the cut; for the diffusate it is $\Delta M = -aBln \frac{C}{C-1}$. Here B is the separation coefficient previously defined for molecular diffusion or evaporation.

Mr. Herzfeld is testing these conclusions experimentally by working with ordinary gases of widely different molecular weights.

88. SEPARATION OF ISOTOPES BY SYSTEMATIC FRACTIONATION.

The equations which have already been given for the process of diffusion may be easily changed so that they are given in terms of the fraction (F) of the material which has passed into the diffused or condensed fractions. Thus for the total diffusate the increase of molecular weight is:

$$(\triangle M)_{TD} = \frac{1 - F}{F} E.\ B\ ln(1 - F)$$

and that for the instantaneous diffusate (ID) is:

$$(\triangle M)_{ID} = (\triangle M)_R - EB = EB\,[-\ ln(1 - F) - 1].$$

The total difference in molecular weight between the two fractions produced ($\Delta\Delta M$) is:

$$(\triangle\triangle M) = -\frac{EB}{F}\ ln(1 - F).$$

Mulliken has recently used these equations to add to the theory of the systematic fractionation of isotopes already presented in Section 78, and a few of the principal results will be outlined here. In planning an extensive separation procedure it is of great importance to be able to estimate the value (V) of the fractions produced. At the present time the best that can be done is to estimate this value in terms of the time necessary to produce any fraction or set of fractions by diffusion or evaporation, since this is the only general method which has proved successful. It is found that the value of *any fraction* is proportional to the volume (or number of mols) to the square of the difference between its molecular weight and that of the initial material, inversely proportional to the square of the separation coefficient (B), and is therefore proportional to the square of the efficiency of the process. This may be formulated as follows:

$$V = \left(\frac{\triangle M}{B}\right)^2 \cdot Q = \triangle^2 Q, \text{ where } \triangle^2 = \left(\frac{\triangle M}{B}\right)^2$$

since ΔM varies as the efficiency (E), the value obtained varies as E^2. This relation holds only when the diffusate is collected in infinitesimal fractions. There is a progressively increasing loss of value as the size of the fractions of the distillate collected is increased. Thus if the fraction in the distillate is 0.1, there is a loss of 0.1 per cent. of the *value* which is almost negligible. The following table illustrates these losses:

TABLE XI.V.

Loss of Value in Collecting Finite Instead of Infinitesimal Diffusate Fractions.

Fraction in Diffusate = F.	Per cent. Loss of Value for Diffusate and Residue.
0.0	0.0
0.1	0.1
0.2	0.5
0.3	1.3
0.4	2.0
0.5	4.0
1.0	100.0

Thus there is a loss of 4 per cent. in the total value for a cut into two equal fractions ($F = 0.5$). This amounts, however, to a loss of 7.7 per cent. of the value of the diffusate, and a zero loss of value in the residue.

The *value* of an *extreme fraction* may be considered proportional to the time required to build up the entire set of intermediate fractions leading to the desired extreme, and is proportional to the e'th power of the separation when $F = 0.5$, and approaches the cube of the separation as the fraction in the diffusate becomes smaller.

The rapidity of the attainment of a given extreme separation depends on how much of the value is allowed to remain in the intermediate fractions, and this is larger when the fraction in the diffusate is 0.5, than when infinitesimal fractions are collected, since in the former case the fractions of volume Q' need to be spaced intervals of EB *ln* two units of atomic weight, while with infinitesimal fractions the same volume Q' is distributed uniformly over a large range of EB units of atomic weight. However, notwithstanding its advantages the method of infinitesimal fractions is impracticable, and for general use the cut of two, that is $F = 0.5$, seems to give the best general plan of procedure, *especially since it leads to a symmetry of the fractions.*

In general the best design for an apparatus is that which gives a maximum for E^3R, in which E is the fractional efficiency and R is the rate of diffusion (or evaporation).

The time necessary to produce a given change of atomic weight is equal to approximately the e'th power of the separation coefficient, provided the diffusion rate and the efficiency can be kept equal in the different processes. Thus a given change in the

atomic weight of zinc could be obtained in one-twenty-eighth (1/28) the time necessary with mercury, provided the zinc could be manipulated as simply as the mercury.

In the process of an extended fractionation many fractions are obtained, and it is important to know the loss of value which occurs when they are mixed in order to decrease the inconvenience of handling, which becomes great if this is not done. This loss of value is:

$$-\delta V = (\triangle_1 - \triangle_2)\frac{Q_1 Q_2}{Q_1 + Q_2}$$

If two equal quantities of mercury with a difference of density of 6.3 parts per million are mixed, the loss of value is about 2 per cent. of that obtained in an operation which divides the material into two equal portions. If the difference in density is 10.5 p.p.m. this loss is 5.8 per cent. In general all fractions which do not differ by more than 7 p.p.m. should be mixed in the case of mercury.

In the diffusion of mercury vapor through the walls of a tube of filter paper it was found that the rate (R) of diffusion varies directly as about the 1.3 power of the length of the tube, and inversely as about the 1.3 power of the diameter. Thus as the area of the diffusion membrane was *increased* by increasing the diameter of the tube, a seemingly contradictory result was obtained, which was that the total amount of material diffusing through it decreased. This was because the mercury vapor was supplied to the tube at a given rate, and the greater pressure in the smaller tubes more than made up for the smaller diffusion area.

89. THE DISCOVERY OF HAFNIUM.

Five elements of atomic numbers 43, 61, 75, 85, and 87 remain unknown. This is of interest and in accord with the idea of the writer that the odd-numbered elements are less abundant and, in general, less stable than those which are even. In this connection it is important to note the recent discovery of element 72, the last of the elements of even atomic number to be found. In Fig. 24, Section 50, this element is listed as celtium, the last of the rare earths, according to the announcement of its discovery by Urbain and Dauviller. In *Nature* of January 20, 1923, Coster and Hevesy announce the discovery of this element, which they name hafnium, in zirconium minerals, from which they

assume that it belongs to the same group as zirconium. This is in better accordance with the form of the periodic table given in Fig. 24.[132] The element hafnium was discovered by means of its X-ray spectrum, according to the method discovered by Moseley. The lines of the element which were measured belong to the L series and have wave-lengths as follows: $L_{\alpha_1} = 1.565$, $L_{\alpha_2} = 1.576$, $L_{\beta_1} = 1.371$, $L_{\beta_2} = 1.323$, $L_{\beta_3} = 1.350$, $L_{\gamma_1} = 1.177$ Ångstrom units. The relation of these lines to those of other elements is shown in Fig. 57.[133]

ERRATA.

1. In Fig. 36 the curve for mercury should be at half the height for that of CCl_4.

A Study of the Deformation of the Photographic Film. CARL BENEDICKS. (*Comptes Rendus*, Oct. 30, 1922.)—In photographing the total eclipse of the sun in 1914, by a mistake the sun's image appeared near the edge of the plate. Later a crack developed at the edge and moved inward until it reached the image of the sun, when it thereupon followed the circular contour. "If, as this unexpected fact makes clear, the light of the sun can cause so considerable an effect, it may be feared that it can cause also a deformation of the sensitive film that cannot be neglected when one is dealing with measurements as delicate as those necessary to be made in order to verify the deviation of light in traversing the gravitational field of the sun." To see whether such a deformation is really caused the following experiment was devised. A glass plate was exposed to powdered bitumen. It was then heated to assure the adhesion of the particles to its surface. It was then photographed. The negative thus obtained had a multitude of sharply outlined white disks. From this a positive was made at the centre of which an artificial sun was produced by letting a circular beam of light fall. This positive represented the plate on which the crack formed and on which distortion might well be produced by the presence of the fictitious image of the sun. Careful measurements showed that no deformation greater than .002 mm. was present. With an objective of focal length equal to 19 feet, such as that used at Sobral in 1919, this would give .07 sec. of arc as the maximum error in the position of a star due to film distortion.

G. F. S.

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[132] It may be seen that the remnant of the disc upon which element 72 was marked, is still present in Fig. 24, to the right of the symbol Ta. See McCoy and Terry's "General Chemistry," page 565, for the proper position for element 72, provided Coster and Hevesy's announcement is correct.

[133] *J. Soc. Chem. Ind.*, 1923, **42**, 67.

First Attempt to Teach Conservation of All Natural Resources of a State.—A forward step in the vital matter of conserving our natural resources has been taken in the preparation of an educational handbook of the resources of Pennsylvania. This book, prepared for use in the Pennsylvania schools to carry out the Smithsonian Institution's purpose of the "increase and diffusion of knowledge among men," presents in novel and striking ways the present situation, causes of waste and how to stop it, and future problems in connection with the conservation of resources. The book will be placed in practically every school in Pennsylvania, and by including the resource material in the geography course, it will be possible to instill into the minds of the school children, the citizens of to-morrow, the necessity of understanding and conserving the resources provided by Nature. While the present handbook is limited to the State of Pennsylvania, there is a nation-wide lesson in it, and such material brought together and placed in the schools of every State would be of immeasurable benefit to the present and future economic condition of the country.

Pennsylvania's primary resource is coal. In this one State is produced 31 per cent. of the country's bituminous and all of the anthracite coal. The chief sources of waste are: Beehive coke-ovens, steam locomotives, which utilize only about 4 per cent. of the energy in coal; heating homes, in which the ordinary coal furnace delivers only about 25 per cent. of the heat energy and unburned coal is dumped into the ash cans. In 200 tests recently made, the average ash can was found to contain 50 per cent. of coal.

In connection with the fullest development of the State's resources there is described the superpower system as proposed by the U. S. Geological Survey. Briefly this system contemplates the standardization of the electrical characteristics of all existing and future electric power plants and transmission lines (in the superpower zone, between Boston and Washington) so as to permit of their interconnection with each other, and so that they will all feed into one huge system of transmission. The great benefits to be derived from this system would appear in the much greater amount of power made available and in a saving of fifty million tons of coal annually.

H. L.

Chemistry as an Aid in the Detection of Crime.—The first treatise on toxicology or the science of poisons was written by Orfila and was published during the first quarter of the nineteenth century. Since that time, various chemical methods have been devised, and then utilized for the detection of crime. These methods are reviewed by HENRY LEFFMANN (*Am. Jour. Pharmacy*, 1922, xciv, 691–699). The precipitin test is specific for the blood of man and certain man-like apes. Crystallographic methods serve for the identification of various compounds. Special procedures are used to detect the adulteration of foods and beverages. Physical and chemical tests reveal alterations in, and forgeries of, documents. J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

FORMULA FOR THE RATE OF EXHAUSTION OF A LARGE TANK BY A RECIPROCATING AIR PUMP.[1]

By E. Buckingham.

[ABSTRACT.]

Conditions.—The conditions for the validity of the formula are as follows: (1) The piston and valves do not leak; (2) there is no appreciable throttling except at the valves; (3) the volume to be exhausted is very large in comparison with the piston displacement; (4) the temperature of the air in the tank is constant; (5) the temperature in the pump at the end of any suction stroke is constant; (6) the pump discharges to atmospheric pressure, and the air in the tank starts at atmospheric pressure.

Notation.—Let

P = atmospheric pressure;
N = number of completed pump cycles;
p = pressure in tank after N cycles;
$x = p/P$ = degree of exhaustion;
l = limiting or lowest attainable value of x;
a = fraction of 1 atmosphere required to lift the discharge valve;
b = fraction of 1 atmosphere required to lift the intake valve;
c = ratio of clearance to piston displacement;
t = ratio of absolute temperature in the pump at the end of a suction stroke to absolute temperature in the tank;
v = ratio of volume of tank to piston displacement;
n = exponent in the equation of the compression line, pv^n = constant.

In any one problem the last six quantities are pure numbers and independent of the units used. a, b, and c are always small; if the valves are operated positively, $a = b = 0$; if the pump is oil sealed, $c = 0$. The temperature ratio t never differs much from unity. The volume ratio v is supposed to be large. The exponent n is between 1.0 and 1.4.

* Communicated by the Director.

[1] Technologic Papers, No. 224, price five cents.

Results.—The formulas obtained are:

$$N = A \log_{10} \frac{C}{(x-b)^{\frac{1}{n}} - B}$$

and

$$l = B^n + b$$

where

$$A = \frac{2.303\ ntv.}{1+c}$$

$$B = \frac{c}{1+c}(1+a)^{\frac{1}{n}},$$

$$C = (1-b)^{\frac{1}{n}} - B$$

AMERICAN AND ENGLISH BALL CLAYS.[3]

By H. H. SORTWELL.

[ABSTRACT.]

THE twenty-one ball clays being used in the largest quantities in the manufacture of china, semiporcelain, electrical porcelain, sanitary ware, and floor and wall tile were studied at the Bureau of Standards. Determinations were made of the water of plasticity and shrinkage relations upon drying, amount of arenaceous material retained by 120-mesh sieves, rate of flow of slips, cohesion of the plastic clay, dry transverse strength when mixed with an equal part of flint, time required for oxidation of carbonaceous material at 750° C., porosity and volume-changes from Orton Seger cone 01 to cone 12, and coloring effect in a standard body.

No great differences were noted in water of plasticity, all of the clays falling within a range of a few per cent. The Dorset clays showed a slightly higher drying shrinkage than the other clays studied. All of the clays showed a tendency to warp in drying, but none developed cracks.

The American clays were much cleaner than the English and contained less coarse mineral matter and dirt which would be removed in lawning the body after blunging. The Kentucky clays were almost free from such material, and the Tennessee clays contained only a small amount, while the English clays were less desirable in this respect.

The transverse strength of equal mixtures of the clays with commercial potters' flint was determined to give a better indi-

[3] Technologic Papers, No. 227, price ten cents.

cation of the comparative bonding power of the clays in a body than would be indicated by the modulus of rupture of the clay alone. The average modulus of rupture of the Tennessee clays tested was 366 pounds per square inch, the Kentucky clays averaged 282 pounds per square inch, the clays from Dorset, England, 405 pounds per square inch, the Devonshire clays 443 pounds per square inch, and English clays, whose exact source was not known, 419 pounds per square inch. In general the English clays were stronger than the domestic, but two of the American clays compared favorably with the average English clay.

From the results obtained the rate of flow of slips from the efflux viscosimeter did not appear to be a valuable criterion of the inherent plastic qualities for use in comparing ball clays. Soluble salts in the clays probably affect the results.

The cohesive strengths of the plastic clays were determined at different water contents and are shown graphically. These results showed no distinctions which compared with the value of the clays for jiggering as judged by practical potters.

Some of the English clays are very high in carbonaceous matter, eleven hours being required in two cases to completely remove the black core from specimens 1¼ by 1¼ by 2 inches. The clays from Devonshire as a class required the longest time for oxidation. The Dorset clays and those from Tennessee contained a moderate amount of carbonaceous matter, while the Kentucky clays contained but little. It was noticed that there was a relationship between the amount of carbonaceous matter and the strength when dry, the more carbonaceous clays usually being stronger.

The burning behavior of the American and English clays was radically different. The English clays vitrified at a low temperature and remained almost constant in porosity and volume up to cone 12. No evidence of overburning was noticed at that temperature. There was some variation in the burning behavior of the clays from Devonshire, but the Dorset clays underwent practically the same changes in firing. The American clays showed a gradual reduction in porosity from cone 01 to cone 12. Tennessee ball clay No. 5 was the only domestic clay which vitrified at cone 8. Tennessee No. 3 matured at cone 10, and Tennessee No. 11 and Kentucky No. 4 at cone 12. The other two Kentucky clays studied were not vitrified at cone 12. Throughout the firing

range studied the American clays were constantly changing in volume.

In the standard body the Tennessee clays as a class showed the best color. The Kentucky and Devon clays were fair in this regard while the Dorset clays were comparatively poor in coloring effect.

Although the English clays have a more desirable firing behavior and greater strength when dry, the American clays contain less material to be removed in body preparation, contain less carbonaceous matter which may lead to trouble in firing, and burn to a better color in the body. Because of their better color they may be used in larger quantities, thereby overcoming in a measure their lower strength. With a slight adjustment in flux content of the body they may be successfully used to replace the English clays.

All of the results are given in tabular form and graphically when feasible. A detailed description of each individual clay is given, and a classification of ball clays based on their properties has been drawn from the results of the work.

The Rotation of the Plane of Polarization of Electromagnetic Waves Produced by a Three-dimensional Grating. K. F. LINDMAN. (*Ann. der Phys.*, vol. 69, p. 270, 1922.)—A few years ago this Finnish physicist showed by experiment that when plane polarized Hertzian waves pass through a system of spiral electrical resonators their plane of polarization is rotated in accordance with a theory advanced by Drude. Later he found further that an assemblage of a large number of left-wound resonators, arranged at random in space, and thus constituting a three-dimensional grating, would rotate to the right or to the left the plane of the polarized wave passing through, according to whether the wave-length of the incident wave was longer or shorter than that of the wave corresponding to the resonators.

In this paper he tests a formula given by Natanson for the case just described, and finds excellent agreement. The wave-length of the incident waves was varied so as to be in part longer and in part shorter than the characteristic wave-length of the resonator, 19.2 cm. The following list gives first the wave-length of the incident electrical radiation, then the angle through which the plane was rotated upon passage through the space grating; 34.2 cm., 5°; 26 cm., 11°; 22.8 cm., 16°; 20.0 cm., 10°; 19.2 cm., 0°; 15.4 cm., −22°; 12.2 cm., −16°.

G. F. S.

NOTES FROM THE RESEARCH LABORATORY EASTMAN KODAK COMPANY.*

FILM DISTORTION AND ACCURACY OF PHOTOGRAPHIC REGISTRATION OF POSITION.[1]

By F. E. Ross.

THE results of the investigations by Schlesinger, Perrine, and Albrecht are summarized. In this investigation photographs of artificial stars were made with a precision camera reducing in the ratio of 1 to 20, on plates one inch wide pressed firmly against a rigid metal frame. In agreement with previous results, no evidence of systematic errors in distance was found, indicating that, in general, distortions are of purely local origin and extent. While the average measurements of ten exposures on a triple-coated plate are practically the same as for ten exposures on a thin emulsion, the local distortions are greater on the thick film. Yet the greatest variation from the mean was only 0.0044 mm. Faint and normal exposures gave the same average measurements within 0.0003 mm. Examination of the images showed that the effects of local variations in distribution of the silver bromide grains and of variations in sensitiveness and in development are negligible. Intensification modifies the minor details of image structure, but does not remove the larger irregularities in the original images nor change the centre of gravity appreciably. Measurements of the same plates, when wet and dry, agree accurately and indicate that, on drying, the images move perpendicular to the surface. However, this is not the case for images within a few millimetres of the edge, as these move outward on drying with alcohol or by hardening with formalin. It is concluded that the principal factor in producing the slight image displacements observed is local non-homogeneity of the gelatin. The probable error in accuracy of measurements of the position of stellar images is found to be considerably less in the case of plates with a fine grain thin emulsion (astronomical) than for those with thicker emulsions (Seed 30 and triple-coated). Intensification of weak

* Communicated by the Director.

[1] Communication No. 154 from the Research Laboratory of the Eastman Kodak Company and published in *Astrophys. J.*, January, 1923, p. 33.

images also decreases the error. In the foregoing measurements the probable error of a single setting was about 0.0005 mm. whereas displacement due to film distortion varied up to 0.002 mm. as a maximum, but averaged only 0.0005 mm. for normal exposures on the thin emulsion.

Food Requirements of Man.—The Food (War) Committee of the Royal Society has published a report on the food requirements of man and their variations according to age, sex, size, and occupation. On account of refuse and waste, the food as purchased should contain approximately 10 per cent. more utilizable calories than are actually required by the individual. The energy requirement of a man varies with the work performed, and ranges from 2500 to 5000 calories daily; it may rise as high as 6000 or even 8000 calories daily in exceptional cases such as the Canadian lumbermen in winter. The average working woman requires 2400 calories of energy daily; women engaged in sedentary labor, such as typewriting, require only 1900 calories daily. Brain workers require from 2200 to 2600 calories daily; their diet should be relatively high in protein. During exposure to low temperatures, extra energy should be supplied as protein (meat or fish) to those engaged in sedentary occupations, and as fat to those engaged in bodily work. Children under six years of age have 50 per cent. of the food requirement of an average man, those between six and ten years 60 per cent., and those between ten and thirteen years 83 per cent. of that requirement; those over thirteen years of age require the same amount of food as an adult of the same sex. A growing child working with the same energy as an adult man requires approximately 200 additional calories daily.

The diet of an average man should contain at least 70 or 80 grams of protein daily, a portion of which should be of animal origin. Milk should serve as a source of protein in the diet of infants and young children. The minimum amount of fat required daily varies with the race, from 20 grams in the case of a Japanese soldier to 75 grams in the case of people of British descent. From 35 to 40 per cent. of the total energy requirement of the body may be supplied as fat; those engaged in vigorous muscular work should receive at least 25 per cent. of their energy intake in the form of fat. A certain amount of fresh fruits or green vegetables should be eaten to supply the requisite vitamins. The diet should also include carbohydrates and salts.

A condition of semi-starvation, with the food intake reduced to two-thirds its normal value, apparently can be endured for a period of several months without danger to health, but gives rise to a greatly reduced resistance to tuberculosis if extended over a period of several years. J. S. H.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

CONPHASEOLIN.[1]

A NEW GLOBULIN FROM THE NAVY BEAN, PHASEOLUS VULGARIS.

By Henry C. Waterman, Carl O. Johns and D. Breese Jones.

[ABSTRACT.]

FRACTIONATION of a 2 per cent. aqueous sodium chloride extract of the navy bean, *Phaseolus vulgaris*, with ammonium sulphate as the precipitant, separated the protein content of the extract into two principal fractions. One of these was identified with the phaseolin previously described by Osborne, but the other the α-fraction in ammonium sulphate precipitation, proved to be a hitherto unknown globulin, having a chemical composition which differentiated it sharply from both of the proteins (*phaseolin* and *phaselin*) isolated by Osborne. The name *conphaseolin* has been assigned to the new substance. It is distinguished from phaseolin and phaselin by its very much higher sulphur content. In nitrogen and in carbon content also, however, as well as in certain of its physical properties, it is notably different from either of the two proteins with which it is associated.

Conphaseolin is an α-globulin of the same type as those previously found in the lima, mung and adsuki bean. It has the typical general properties of the group. It is precipitable by ammonium sulphate at a relatively low concentration, is readily denaturable, and possesses a high carbon content and a very high sulphur content. The lysine figure, 10.69, is the highest which has thus far been found for any vegetable protein.

CHRONIC INTOXICATION BY SMALL QUANTITIES OF CADIUM CHLORIDE IN THE DIET.[2]

By Carl O. Johns, A. J. Finks and Carl L. Alsberg.

[ABSTRACT.]

VARIOUS concentrations of cadmium chloride were incorporated in a diet which was known to be adequate for the normal growth of rats. Diets containing from 1000 to 62.5 parts per million of cadmium were tested. Very little or no growth

* Communicated by the Chief of the Bureau.

[1] Published in *J. Biol. Chem.*, Feb., 1923, **55**, 93.

[2] Published in *J. Pharmacol.*, Feb., 1922, **21**, 59.

occurred and death ensued when the concentration of cadmium was 1000, 500, or 250 parts per million. When there were 125 parts per million of cadmium in the diet, the initial rate of growth was normal. All of the male rats receiving this concentration of cadmium died in about fifty days, while the majority of the females survived a very much longer time, one of them dying at the end of 199 days, while another was still living at the end of 280 days.

A concentration of cadmium of 62.5 parts per million had no effect on growth, the rate of growth and food intake being normal. The food intake increased as the concentration of cadmium in the diet was decreased. Whether the interference with growth was due solely to the diminished food intake was not determined. With a dosage of 0.56 mgm. of cadmium per day no cumulative action was observed.

STUDIES ON THE PHARMACOLOGY OF CADMIUM AND ZINC WITH PARTICULAR REFERENCE TO EMESIS.[3]

By Erich W. Schwartze and Carl L. Alsberg.

[ABSTRACT.]

ZINC, calculated as the metal, is about equally toxic for all species studied (rats, rabbits, cats and dogs) when injected intravenously. Cadmium is about three times more toxic to cats and dogs, but to rabbits and rats four or five times more toxic than zinc.

On oral administration the lethal dose of zinc is from five to seven times greater than that of cadmium. In dilute solutions used, cadmium is more toxic than in more concentrated solutions.

The consumption of an average-sized meal of raw hashed meat containing 350 to 400 parts per million of cadmium or 3000 parts per million of zinc is almost always followed by emesis. Cadmium calculated as the metal is eight to nine times more effective as an emetic than zinc.

Concentrations of cadmium of 250 or more parts per million in the diet are incompatible with life. This is also the mean emetic concentration. Recovery followed reduction of this concentration. No evidence of cumulative systematic action was

[3] Published in *J. Pharmacol.*, Feb., 1922, 21, 1.

obtained in these experiments. However, slight loss of appetite, nibbling of food and occasional vomiting indicate that, although apparently compatible with life, the presence in the food of the lower concentrations tried is decidedly objectionable.

No evidence of storage of cadmium other than in the kidney, liver and spleen was obtained. The kidney usually contained the greater part; the spleen contained the least and sometimes none.

Vertical Movements of the Atmosphere with Overcast Sky and the Connection between Cloudiness and the Velocity with which Depressions Travel. G. Reboul. (*Comptes Rendus*, December 4, 1922.)—The existence of ascending currents of air explains very readily the formation of cumulus clouds. On the other hand, there are meteorologists who regard the condensation of water vapor in the higher regions of the atmosphere as the chief cause of ascending currents. No matter which view may in the end prove to be the correct one, it is of interest to investigate experimentally the connection existing between cloudiness and the velocity of the ascending air currents. The usual method of following a balloon in its upward flight by means of a theodolite breaks down in cloudy weather because the clouds interfere with the sight of the balloon. The Geographical Service of the French Army toward the end of the war had established stations for exploring the air in cloudy weather. For some months such a station was conducted by Mr. Reboul, who availed himself of the opportunity to investigate the question at issue. Acoustical methods were used to indicate the successive positions of the ascending balloons, which usually attained an elevation of from 5000 to 6000 metres. In all, 270 ascensions of small balloons were made. In working up the results comparison was made between the velocity of ascent under a certain kind of sky and the normal velocity, allowance being made for the size of the balloon employed. "These results show very definitely that, when the sky is clear or only slightly cloudy, the balloon mounts with a velocity less than the normal. On the other hand, when the sky is overcast or very cloudy the balloon ascends with a velocity greater than the normal. In this case there are surely ascending currents. When there is rain the effect of the ascending currents is opposed evidently by the shock of the drops on the balloon and by the additional mass of the water adhering to it."

According to Helmholtz, when there are ascending currents, atmospheric depressions can travel more quickly. Applying the results just given, it should likewise follow that atmospheric depressions travel more quickly in cloudy than in clear weather. This conclusion is confirmed by an examination of the quotients got by dividing the mean velocity of depressions in Western Europe in each season of the year by a number designating the cloudiness at

Paris during that season. The four quotients are close together, thus showing that a high velocity of travel is likely to be found when there is much cloudiness. The importance of this relation in predicting the weather is obvious. G. F. S.

Cathode Disintegration. THE RESEARCH STAFF OF THE GENERAL ELECTRIC COMPANY, LTD., London. (*Phil. Mag.*, Jan., 1923.)—In order to minimize difficulties in the interpretation of results the discharge in a triode with a thermionic cathode was used. The anode was a nickel plate, the cathode a tungsten spiral heated by a 6-volt battery, while the grid, lying between them, consisted of a close spiral of fine tungsten wire. "The current to the grid, when the grid is negative to the cathode, is then a measure of the number of positive ions arriving at it, and the speed of the ions arriving is related comparatively simply to the potentials of the electrodes."

The loss of mass in the grid was determined by measuring its increase of resistance due to disintegration. "A change in conductivity amounting to one three-thousandth part of the grid in its initial condition could be detected; this corresponds to a loss of weight of about .0024 mgm."

It was found that for disintegration to take place positive ions must strike. When the grid was an anode, it lost no mass at all. Furthermore, the temperature of the grid makes no difference to the disintegration, at least within the range 200° to 1200° C.

The relation between the number of atoms of tungsten lost and the number of positive atoms striking the grid was calculated. This ratio is always a proper fraction, in a few instances exceeding .5. With the same difference of potential maintained in all cases between the anode and the grid this ratio varies greatly with the nature of the surrounding gas. It is least with hydrogen, then follow in ascending order, helium, nitrogen, a mixture of helium and neon, mercury vapor and argon. The highest value found for it was about 100 times the smallest.

In all cases, except with argon, disintegration is accompanied by a decrease in the pressure of the gas. "The gas disappears slowly, being absorbed in, or on the surface of, the metal deposited on the walls." "Nitrogen probably disappears most rapidly and helium least rapidly." G. F. S.

A New Radioactive Mineral, Parsonite. A. SCHOEP. (*Comptes Rendus*, January 15, 1923.)—With the compact green masses of chalcolite from the Belgian Congo there are found several uraniferous minerals of yellowish color. Along with these an additional mineral has recently been recognized. It occurs in small, pale-brownish crystals of specific gravity 6.23 and of index of refraction 1.99. Its composition seems to be $2PbO.UO_3.P_2O_5.H_2O$. The radioactive constituent, UO_3, forms 32.59 per cent. of the whole. The mineral has been named after Professor Parsons of the University of Toronto. G. F. S.

NOTES FROM THE U. S. BUREAU OF MINES.*

LIGNITE CARBONIZING EXPERIMENTS.

By W. W. Odell.

As the result of experiments conducted by the Bureau of Mines and the University of North Dakota, a lignite carbonizer, simple in construction and operation, and with a relatively very low first cost, has been devised. This oven was developed to fill the need for an oven which can be cheaply built and intermittently operated by lignite mine owners, yielding a char which can, if desired, be used as fuel without further processing.

By-products—gas and tar—are not recovered in the operation of this carbonizer, but with certain alterations in design they may be recovered, although they probably will not be as valuable as the by-products from other carbonizers are usually rated. The carbonizer may be operated intermittently without serious damage to it. Lignite char can be made at a much lower cost (approximately $1.25 per ton less) in this oven than in the by-product ovens hitherto suggested for carbonizing lignite. This is chiefly due to the large capacity and low cost of the carbonizer. Details regarding this lignite carbonizer are given in Serial 2441, of the Bureau of Mines, Department of the Interior.

Following the favorable tests on lignites of North Dakota, by the Bureau of Mines in coöperation with the University of North Dakota, arrangements were made for operating the vertical carbonizing oven at Grand Forks, North Dakota, using Canadian lignite. The present work is being done in coöperation with the Canadian Lignite Utilization Board, in order to determine whether the oven is suitable for the treatment of Canadian lignites. A large-scale test of Canadian lignite was so satisfactory that the board has decided to erect a Bureau of Mines oven at its experimental plant at Bienfait, Saskatchewan.

CONDENSATION LOSSES OF WATER GAS.

By W. A. Dunkley.

THE Central District Experiment Station of the Bureau of Mines at Urbana, Illinois, recently undertook a study of condensation losses due to the transmission of carburetted water gas

* Communicated by the Director.

under high pressures in the distribution system of the Western United Corporation of Aurora, Illinois. The most distant distributing point on the system is at Harvard, Illinois, about eighty-five miles from the plant. The territory supplied is approximately eighty-five miles long and thirty-five miles wide, the total mileage of mains being 1223, or approximately 24 per cent. of all the gas mains in Illinois outside the city of Chicago. With so extensive a distribution system and so high an initial pressure, it is not surprising that the unaccounted-for gas, or excess of production over sales, should be higher than in a small low-pressure system. In a fairly good system of the latter type, the excess of production over sales is seldom less than 5 or 6 per cent. and instances are by no means uncommon where the unaccounted loss is double this figure. In the Western United System the unaccounted-for gas for a time was in the neighborhood of 20 per cent. Details of the investigation are given in Serial 2447 of the Bureau of Mines.

YIELD AND QUALITY OF BY-PRODUCTS FROM FREEPORT COAL.

By Joseph D. Davis.

A STUDY of the yield and quality of the gas, oil and other constituents of the Freeport coal-bed in Pennsylvania has been completed by the U. S. Bureau of Mines in coöperation with the Carnegie Institution of Technology and with mine operators and engineers. This investigation was undertaken in order to obtain data on the possibility of recovering from the cannel and bone of this coal-bed, oils and other by-products as can be done with oil shales. The tests show that the cannel coal yields 27 to 33 gallons of tar oils, 3800 to 4500 cubic feet of gas, 9 to 10 pounds of ammonium sulphate, and 75 per cent. of a rather high-ash smokeless fuel per ton of raw material carbonized at low temperatures. It is possible that in the relatively near future this cannel coal, now wasted, can be commercially treated at a profit, but it is doubtful whether bone coal can be so treated. The time when utilization will become economical depends on the rate of diminution of petroleum resources. The complete details have been published as Bulletin 1 of the Carnegie Institution of Technology.

SPONTANEOUS COMBUSTION OF BITUMINOUS COAL.

By Joseph D. Davis.

Safe storage of bituminous coal, which is probably the only solution of stabilizing the coal industry, involves means to overcome hazards of spontaneous ignition when coal is stored in large piles. On account of the divergence of opinions and the variety of methods proposed for storing coal, the Bureau of Mines, in coöperation with the Carnegie Institution of Technology, has conducted a laboratory study of the factors involved in spontaneous combustion of bituminous coal. One of the principal objects was to determine why some coals ignite more readily than others.

A laboratory test was devised for determining the relative tendency of coals to fire spontaneously, and this "critical temperature" test was applied to many coals. The conclusions reached as a result of this study are that fine coal dust and moist air quickly increase the hazards of spontaneous combustion. Coal finer than 200 mesh has a critical temperature of 171° C., while coal between 10 and 20 mesh has a critical temperature of 231° C. Coal coarser than one-fourth inch showed no rapid self-heating. Therefore, the presence of fines in a coal pile should be avoided. Moisture in the air or coal facilitates oxidation and consequent heating. Pyrite in lump form has no dangerous effect in coal.

Mixtures of fresh and partly oxidized coal heat no more rapidly than fresh coal. The "anthraxylon" or "woody" constituents of the coal have a lower critical temperature than the "attritus" constituents, but more investigation is needed to evaluate the readily oxidizable constituents in coal and thus determine which are the more responsible for spontaneous heating. The critical-temperature method evolved for testing coals seems suited for grading their relative tendencies to fire spontaneously, and the Bureau of Mines expects to develop this method further and apply it in a survey of the coals of the United States. Further details will be found in the recent report on this subject, which was published as Bulletin 3 of the Carnegie Institution of Technology.

The Evaluation of the Colors of the Spectrum in Terms of the Three Primary Colors. R. A. Houstoun. (*Phil. Mag.*, Jan., 1923.)—"If we take three colors—a red, a green, and a blue—and represent them by the corners of an equilateral triangle, then any

color whatever can be represented by a point in the plane of the triangle. Any color obtained by mixing two colors is represented by a point on the straight line joining these two colors which divides the distance between them inversely in the ratio of their intensities. That is, to find the position of the mixture we regard the colors as particles the masses of which are proportional to their intensities, and proceed to find the centroid of these particles." All the colors that can be obtained by combining in any proportions the three primary colors have locations within the triangle.

Determinations of the composition of colors according to this plan have been made by several experimenters. Four years ago Doctor Houstoun worked over their results so as to express them in the same units and found that considerable divergences existed. He has now devised a new method of comparison and has made an independent series of determinations. He has learned, moreover, why his predecessors failed to agree among themselves. "As a result of our investigation, we are of the opinion that this is because the subject does not admit of very exact measurement; besides the differences which occur between different observers, the measurements made by the same observer vary somewhat from time to time." "Of two determinations made on different days of the same week one agreed closely with the position given on the diagram, and the other was almost twice as far out from the corner of the triangle. As the observations were made with the greatest of care and under exactly the same conditions on each occasion, and similar differences had been noticed on former occasions, there seems no doubt that the color-vision changes appreciably from day to day." G. F. S.

Application of an Optical Manometer to the Measurement of the Viscosity of Gases. J. E. P. WAGSTAFF. (*Phil. Mag.*, Jan., 1923.)—A thin slip of glass of known modulus of elasticity forms the upper wall of a pressure chamber. Upon it rests a plate of glass one millimetre thick. The film of air between the slip and the plate is illuminated by a parallel beam of monochromatic light. The circular fringes there formed are viewed by a microscope. Any bulging of the thin slip will cause a radial motion of the fringes. The pressure chamber is connected to a partially exhausted reservoir, into which air may slowly flow from the outside atmosphere through a capillary tube. As the air enters, the pressure within both reservoir and pressure chamber rises and the glass slip bulges upward, causing the fringes to migrate in toward the centre. In one instance it took 43 seconds for 10 fringes to disappear, the pressure meanwhile rising through 18.1 cm. of mercury. There is, of course, a relation between the dimensions of the apparatus, the elastic coefficient of the glass slip and the viscosity. This last quantity was calculated from the data of the experiment to be 1.811×10^{-4} dynes per sq. cm. per unit velocity gradient. G. F. S.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting held Wednesday, March 21, 1923.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, March 21, 1923.

MR. W. C. L. EGLIN, Vice-president, *in the Chair.*

The Board of Managers submitted its report. The report recorded the election to membership of:

Christo Casacof, Esq., Mechanical Engineer, 104 West 79th Street, New York City, New York.

Henry Walter Cockerill, Esq., Technical Clerk, 19 Rata Road, Hataitai, Wellington, New Zealand.

Antonius de Graaff, Esq., Chemical Engineer, Philips' Glowlampworks, Eindhoven, Holland.

John F. Metten, Esq., Chief Engineer, William Cramp and Sons Ship and Engine Building Company, Richmond and Norris Streets, Philadelphia, Pennsylvania.

Allen I. Myers, Esq., A.B., Instructor in Chemistry, Swarthmore College, Swarthmore, Pennsylvania.

Charles C. Roberts, Esq., Chemist, 75 W. Lacrosse Avenue, Lansdowne, Pennsylvania;

lectures before the Sections as follows:

"The Structure and Constitution of Alloys" by
Walter Rosenhain, D.Sc., F.R.S.,
Superintendent, Metallurgy Department,
National Physical Laboratory,
Teddington, England;

"High Temperature Investigation" by
Edwin F. Northrup, Ph.D.,
Princeton,
New Jersey;

"Ionization and Resonance Phenomena" by
Charles B. Bazzoni, Ph.D.,
Professor of Experimental Physics,
University of Pennsylvania,
Philadelphia, Pennsylvania;

and a lecture before the Stated Meeting, February 21, 1923, by
Clement E. Chase, C.E.
Principal Assistant Engineer
and
M. B. Case, B.Sc.,
Senior Resident Engineer,
Delaware River Bridge Joint Commission,
Philadelphia, Pennsylvania,

on "The Main Piers of the Bridge over the Delaware River between Philadelphia and Camden;"
also the following additions to the library: 38 bound volumes, 8 unbound volumes, 115 pamphlets and 7 photographs.

The Board of Managers also transmitted the following resolutions:

Resolved, That the Board of Managers of The Franklin Institute recommend to the Institute the election to Honorary Membership of Professor A. A. Michelson, Professor of Physics, the University of Chicago, Chicago, Illinois, who on the third of January was awarded the Institute's Franklin Medal "in recognition of his contributions of fundamental importance in physical science, especially his brilliant discoveries in the field of optics and astro-physics,"

Resolved, That the Board of Managers of The Franklin Institute recommend to the Institute the election to Honorary Membership of General G. A. Ferrié, Chief Signal Officer of the French Army, Member of the French Academy, who on the third of January was awarded the Institute's Franklin Medal "in recognition of his long-continued and successful researches in the field of radio-transmission of intelligence and their splendid and successful military applications, and his eminent success in the organization and directing of the communication service of the French Army during the war."

On motion duly seconded, the report was accepted and the recommendations to elect to Honorary Membership Dr. A. A. Michelson, of Chicago, and General G. A. Ferrié, of Paris, France, were unanimously adopted.

The paper of the evening was presented by Dr. Harvey C. Hayes, Research Physicist, U. S. Navy, on "Measuring Ocean Depths by Acoustical Methods." The measurement of distances by acoustical methods and in particular with the measurement of submarine distances and the determination of submarine contours were first considered. The history of the subject was briefly outlined and the various devices that have been developed in connection with the problem were described. The research work carried out by the Navy Department during the past five years which has resulted in solving the problem of measuring submarine distances through the development of the "Sonic Depth Finder" were reviewed. Contour charts of the sea-bottom to depths exceeding three thousand fathoms were shown and certain possible interesting and valuable applications of the depth finder were pointed out. The subject was illustrated by lantern slides.

After a discussion, a unanimous vote of thanks was extended to the speaker.

Adjourned.

R. B. OWENS,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 7, 1923.)

HALL OF THE INSTITUTE,
PHILADELPHIA, March 7, 1923.
MR. W. H. FULWEILER *in the Chair.*

The following reports were presented for final action:

No. 2763: Universal Measuring Machine. The Edward Longstreth Medal to The Geneva Society of Geneva, Switzerland.

No. 2806: Literature. The Howard N. Potts Medal to Dr. Albert W. Hull, General Electric Company, Schenectady, New York, for his paper on "The Crystal Structure of the Common Elements," printed in the issue of the JOURNAL OF THE FRANKLIN INSTITUTE for February, 1922.

The following report was presented for first reading:

No. 2794: Smoot Regulating System.

R. B. OWENS,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, March 14, 1923.)

RESIDENT MEMBERS.

MR. JOHN F. METTEN, Richmond and Norris Streets, Philadelphia, Pennsylvania.

MR. ALLEN I. MYERS, Swarthmore College, Swarthmore, Pennsylvania.

MR. CHARLES C. ROBERTS, 75 West Lacrosse Avenue, Lansdowne, Pennsylvania.

NON-RESIDENT.

MR. CHRISTO CASACOF, 104 West 79th Street, New York City, New York.

MR. H. W. COCKERILL, 19 Rata Road, Hataitai, Wellington, New Zealand.

MR. ANTONIUS DE GRAAFF, Philips' Glowlampworks, Eindhoven, Holland.

CHANGES OF ADDRESS.

MR. L. J. R. HOLST, Harvard Road, Brookline, Delaware County, Pennsylvania.

MR. A. DW. SMITH, 1732 Commercial Trust Building, Philadelphia, Pennsylvania.

MR. SAMUEL SPITZ, 2244 North Vanpelt Street, Philadelphia, Pennsylvania.

NECROLOGY.

Mr. Calvin Pardee, 447 Drexel Building, Philadelphia, Pennsylvania.

PUBLICATIONS RECEIVED.

Die Luftstickstoffindustrie mit besonderer Berücksichtigung der Gewinnung von Ammoniak und Salpetersäure, von Dr. Ing. Bruno Waeser. 586 pages, illustrations, 8vo. Leipzig, Otto Spamer, 1922. Price, in paper, 13,440 Marks.

Elements de Mécanique à l'Usage des Ingenieurs. Statique cinématique par Robert d'Adhémar. 254 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1923. Price, in paper, 16 Francs.

Index Generalis. General Year Book of the Universities, High Schools, Academies, Archives, Libraries, Scientific Institutions, Botanical and Zoological Gardens, Museums, Observatories and learned societies, published under the direction of R. de Montessus de Ballore. 2111 pages, 12mo. Paris, Gauthier-Villars et Cie., 1922–23. Price, in paper, 50 Francs.

Ultra-violet Radiation—Its Properties, Production, Measurement, and Applications, by M. Lukiesh, Director of Applied Science, Nela Research Laboratories. 258 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, 1922.

Théorie Mathématique des phénomènes Thermiques Produits par la Radiation Solaire, par M. Milankovitch. 338 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1920. Price, 20 Francs.

Cours de Chimie Inorganique, par Fred. Swarts. Third edition, revised and enlarged, 734 pages, illustrations, 8vo. Bruxelles, Maurice Lamertin, 1922. Price, in paper, 50 Francs.

Cours de Chimie Organique, par Fred. Swarts. Third edition, revised and enlarged, 674 pages, illustrations, 8vo. Bruxelles, Maurice Lamertin, 1921. Price, in paper, 50 Francs.

La Théorie de la Relativité d'Einstein et ses Bases Physiques. Exposé élementaire par Max Born. Traduit de l'allemand d'apres la seconde edition par F. A. Finkelstein et J. G. Verdier. 339 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1923. Price, in paper, 25 Francs.

La Théorie de l'Allotropie, par A. Smits. Première edition française traduite par J. Gillis. 523 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1923. Price, in paper, 55 Francs.

Glue and Gelatin, by Jerome Alexander. 236 pages, plate, 8vo. New York, Chemical Catalog Company, 1923. Price, $3.

Our Solar System and the Stellar Universe. Ten popular lectures by Rev. Charles Whyte. 234 pages, plates, 8vo. London, Charles Griffin and Company, Ltd., Philadelphia, J. B. Lippincott Company, 1923.

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